Smooth Skinning Decomposition with Rigid Bones

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Motivation



















• A.k.a. skeleton subspace deformation, enveloping, vertex blending, smooth skinning, bones skinning, etc.





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Skinning Decomposition



Skinning Decomposition



Skinning Decomposition





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• Rigging, animation editing



- Rigging, animation editing
- Compression, hardware accelerated rendering





- Rigging, animation editing
- Compression, hardware accelerated rendering
- Segmentation, meshes simplification





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Skinning Decomposition



Smooth Skinning Decomposition with Rigid Bones

Input: Example poses

Output: Linear Blend Skinning model

- Sparse, convex weights
- Rigid bone transformations
- No skeleton hierarchy



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<u>Goals:</u>

- ✓ Approximate highly deformation models
- ✓ Fast performance
- ✓ Simple implementation



Rigid Bones v.s. Flexible Bones



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Rigid Bones v.s. Flexible Bones

Rigid transformation (orthogonal)

 $\begin{bmatrix} r_{11} & r_{12} & r_{13} & t_1 \\ r_{21} & r_{22} & r_{23} & t_2 \\ r_{31} & r_{32} & r_{33} & t_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ $R^{T}R = I, \det R = 1$

Non-rigid transformation (non-orthogonal)



Animation editing







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Animation editingCollision detection





- Animation editing
- Collision detection
- ✓ Skeleton extraction





- Animation editing
- Collision detection
- ✓ Skeleton extraction



Rigid bones



Flexible bones



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- ✓ Animation editing
- Collision detection
- ✓ Skeleton extraction
- Compact representation

$$(r_1, r_2, r_3, t_1, t_2, t_3)$$
 v.s

.

1	1 1	r_{12}	r_{13}	t_1
1	21	r_{22}	r_{23}	t_2
1	31	r_{32}	r_{33}	t_3
L	0	0	0	1

Rigid bone 6 DOFs Flexible bone 12 DOFs



- Animation editing
- Collision detection
- ✓ Skeleton extraction
- Compact representation





Previous Work

- Cluster triangles with similar deformations to get bones, then optimize skinning weights
 - Skinning Mesh Animations
 [James and Twigg 2005]
 - Example-Based Skeleton Extraction [Schaefer and Yuksel 2007]
 - Automatic Conversion of Mesh Animations into Skeleton-based Animations [de Aguiar et al. 2008]
- X Not consider skin blending, only good for nearly articulated models







Previous Work

- Joint optimize bone transformations and skinning weights
 - Fast and Efficient Skinning of Animated Meshes
 [Kavan et al. 2010]
 - ✓ Linear solvers
 - ✗ Flexible bones
 - Learning Skeletons for Shape and Pose
 - [Hasler et al. 2010] ✓ Rigid bones
 - X Non linear solver

Good approximation of highly deformable models Non-convex optimization, possibly with non-linear constraints

Shape

Combination


Smooth Skinning Decomposition with Rigid Bones

[Le and Deng 2012]

Rigid bones
 Highly deformable models
 Linear solvers



$$\min_{w,R,T} E = \min_{w,R,T} \left[\sum_{t=1}^{|t|} \sum_{i=1}^{|V|} \left\| v_i^t - \sum_{j=1}^{|B|} w_{ij} (R_j^t p_i + T_j^t) \right\|^2 \right]$$

Subject to: $w_{ij} \ge 0, \forall i, j$
$$\sum_{j=1}^{|B|} w_{ij} = 1, \forall i$$

$$|\{w_{ij} | w_{ij} \ne 0\}| \le |K|, \forall i$$

$$R_j^t^\mathsf{T} R_j^t = I, \det R_j^t = 1, \forall t, j$$

$$\min_{w,R,T} E = \min_{w,R,T} \sum_{t=1}^{|t|} \sum_{i=1}^{|V|} \left\| v_i^t - \sum_{j=1}^{|B|} \overline{w_{ij}} (R_j^t p_i + T_j^t) \right\|^2$$
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Subject to: $w_{ij} \ge 0, \forall i, j$
Bone Transformations
$$\sum_{j=1}^{|B|} w_{ij} = 1, \forall i$$

$$|\{w_{ij}|w_{ij} \ne 0\}| \le |K|, \forall i$$

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Subject to: $w_{ij} \ge 0, \forall i, j \longrightarrow$ Non-negativity
$$\sum_{j=1}^{|B|} w_{ij} = 1, \forall i \longrightarrow$$
 Affinity
$$\left| \{ w_{ij} | w_{ij} \ne 0 \} \right| \le |K|, \forall i \longrightarrow$$
 Sparseness
$$R_j^t {}^{\mathsf{T}} R_j^t = I, \det R_j^t = 1, \forall t, j$$

$$\min_{w,R,T} E = \min_{w,R,T} \sum_{t=1}^{|t|} \sum_{i=1}^{|V|} \left\| v_i^t - \sum_{j=1}^{|B|} w_{ij} (R_j^t p_i + T_j^t) \right\|^2$$

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$$R_j^t T R_j^t = I, \det R_j^t = 1, \forall t, j \rightarrow \text{Orthogonal}$$

$$\text{Non-linear}$$

Skinning Decomposition Algorithm

$$\min_{w,R,T} E = \min_{w,R,T} \sum_{t=1}^{|t|} \sum_{i=1}^{|V|} \left\| v_i^t - \sum_{j=1}^{|B|} w_{ij} (R_j^t p_i + T_j^t) \right\|^2$$

Skinning Decomposition Algorithm



Skinning Decomposition Algorithm



Initialization

- No blending (rigid binding): each vertex is driven by exactly one bone
- Assign |V| vertices into |B| clusters
- K-means clustering





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Skinning Weights Solver

• Per vertex solver: Constrained Linear Least Squares

$$W_i^{\mathsf{T}} = \arg\min_x ||Ax - b||^2$$

Subject to: $x \ge 0$
$$||x||_1 = 1$$
$$||x||_0 \le |K|$$



Skinning Weights Solver

• Per vertex solver: Constrained Linear Least Squares

$$W_i^{\mathsf{T}} = \arg\min_x ||Ax - b||^2$$

Subject to: $x \ge 0$ \longrightarrow Bound Constraint
 $||x||_1 = 1$ \longrightarrow Equality Constraint
 $||x||_0 \le |K|$

- Active Set Method [Lawson and Hanson]
 - Pre-compute LU factorization of $A^{\mathsf{T}}A$ and $A^{\mathsf{T}}b$
 - Pre-compute QR decomposition of $\begin{bmatrix} 1 & 1 & \cdots & 1 \end{bmatrix}^{\mathsf{T}}$

Skinning Weights Solver

• Per vertex solver: Constrained Linear Least Squares

$$W_i^{\mathsf{T}} = \arg \min_x ||Ax - b||^2$$

Subject to: $x \ge 0$
$$||x||_1 = 1$$
$$||x||_0 \le |K| \rightarrow \text{Sparseness Constraint}$$

Weight pruning of bones with small contribution

$$e_{ij} = \left\| w_{ij} (R_j^t p_i + T_j^t) \right\|^2$$

Keep /K/ bones with largest e_{ij} and solve the LS again



• Per example pose solver:

$$\min_{R^{t},T^{t}} E^{t} = \min_{R^{t},T^{t}} \sum_{i=1}^{|V|} \left\| v_{i}^{t} - \sum_{j=1}^{|B|} w_{ij} (R_{j}^{t} p_{i} + T_{j}^{t}) \right\|^{2}$$

Subject to: $R_{j}^{t^{\mathsf{T}}} R_{j}^{t} = I$, det $R_{j}^{t} = 1$



• Per example pose solver:

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Subject to: $R_{j}^{t^{\mathsf{T}}} R_{j}^{t} = I$, det $R_{j}^{t} = 1$

 Levenberg-Marquardt optimization
 Optimized solution ×Slow
 Absolute Orientation (a.k.a. Procrustes Analysis) [Kabsch 1978; Horn 1987]
 Fast ×Approximate solution

 Our solution: Solve bone transformation one-by-one to minimize the deformation residual of remaining bones
 ✓ Linear solver, fast, and simple
 ✓ Near optimized solution







































• The residual q_i^t for bone \hat{j}

$$E^{t} = \left\| v_{i}^{t} - \sum_{j=1}^{|B|} w_{ij} (R_{j}^{t} p_{i} + T_{j}^{t}) \right\|^{2}$$

$$E_{j}^{t} = \sum_{i=1}^{|V|} \left\| v_{i}^{t} - \sum_{j=1, j \neq \hat{j}}^{|B|} w_{ij} (R_{j}^{t} p_{i} + T_{j}^{t}) - \underbrace{w_{i\hat{j}} (R_{\hat{j}}^{t} p_{i} + T_{\hat{j}}^{t})}_{\mathbf{Q}_{i}^{t}} \right\|^{2}$$

$$Bone \hat{j} \text{ out}$$



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Bone \hat{j} out

• Now find rigid transformation

$$p_i \xrightarrow{(R_{\hat{j}}^t, T_{\hat{j}}^t)} q_i^t$$



• Remove the translation

$$\overline{p}_i = p_i - p_*$$
$$\overline{q}_i^t = q_i^t - w_{i\hat{j}}q_*^t$$

Center of Rotation:

$$p_* = \frac{\sum_{i=1}^{|V|} w_{i\hat{j}}^2 p_i}{\sum_{i=1}^{|V|} w_{i\hat{j}}^2}$$
$$q_*^t = \frac{\sum_{i=1}^{|V|} w_{i\hat{j}} q_i^t}{\sum_{i=1}^{|V|} w_{i\hat{j}}^2}$$

Calculate the rotation by Singular Value Decomposition

Compare Equations

Remove the translation

Our method	Weighted Absolute Orientation
$\overline{p}_i = p_i - p_*$ $\overline{q}_i^t = q_i^t - w_{ij} q_*^t$	$\overline{p}_i = p_i - p_*$ $\overline{q}_i^t = q_i^t - q_*^t$
Center of Rotation:	Center of Rotation:
$p_* = \frac{\sum_{i=1}^{ V } w_{ij}^2 p_i}{\sum_{i=1}^{ V } w_{ij}^2}$	$p_* = \frac{\sum_{i=1}^{ V } w_{i\hat{j}} p_i}{\sum_{i=1}^{ V } w_{i\hat{j}}}$
$q_*^t = \frac{\sum_{i=1}^{ V } w_{i\hat{j}} q_i^t}{\sum_{i=1}^{ V } w_{i\hat{j}}^2}$	$v_*^t = \frac{\sum_{i=1}^{ V } w_{i\hat{j}} v_i^t}{\sum_{i=1}^{ V } w_{i\hat{j}}}$
Why there are differences?	

Compare Equations

Our methodWeighted Absolute Orientation
$$E^t = \left\| v_i^t - \sum_{j=1}^{|B|} w_{ij} (R_j^t p_i + T_j^t) \right\|^2$$
 $E^t = \sum_{j=1}^{|B|} \sum_{i=1}^{|V|} w_{ij} \left\| v_i^t - (R_j^t p_i + T_j^t) \right\|^2$

Different objective functions!



Compare Equations
























Toy Example



Toy Example



Toy Example



Bone Transformations Re-Initialization

Recall: Center of Rotation

$$p_* = \frac{\sum_{i=1}^{|V|} w_{i\hat{j}}^2 p_i}{\sum_{i=1}^{|V|} w_{i\hat{j}}^2}$$
$$q_*^t = \frac{\sum_{i=1}^{|V|} w_{i\hat{j}} q_i^t}{\sum_{i=1}^{|V|} w_{i\hat{j}}^2}$$



- Improve stability
- Jump out of local minimums

Summary of Our Algorithm





Convergence



• Error decreases monotonically (without re-init bone transformations)

Convergence



- Error decreases monotonically (without re-init bone transformations)
- Our algorithm converges much faster than LM (~50 times)



Convergence



- Error decreases monotonically (without re-init bone transformations)
- Our algorithm converges much faster than LM (~50 times)
- One pass is enough for bone transformations update



Results – Articulated models





Results – Highly deformable models





Rigid Bones vs. Flexible Bones







Dataset[No. of bones]	Approximation error E_{RMS}			Execution time (minutes)		
	SMA	LSSP	SSDR	SMA	LSSP	SSDR
camel-collapse ₁₁	125.3 (4)	-	5.4(1.7)	13.8	-	7.4
cat-poses ₂₅	8.5 (3.1)	6.2(3.3)	3.4(1.4)	0.7	371.7	1.5
chickenCrossing ₂₈	12.5 (4.2)	6.2(5.1)	8.1(5.4)	14.1	1165.4	24
horse-gallop ₃₃	9.5 (1.5)	12.5(4.6)	2.2(1.1)	3.8	911	9.8
$lion-poses_{21}$	62.8 (5.7)	7.7(3.9)	4.4(2.2)	0.6	360.2	0.8
$pcow_{24}$	24.8 (13.2)	7.2(6.7)	5.7(4.8)	3.8	564.5	8.9
pdance ₂₄	3.8 (1.6)	3.4(2.3)	1.3(0.8)	22	2446.8	28.3

SMA: Skinning Mesh Animations [James and Twigg 2005]
LSSP: Learning Skeletons for Shape and Pose [Hasler et al. 2010]
SSDR: Smooth Skinning Decomposition with Rigid Bones (our method) Result in parentheses: rank-5 EigenSkin correction [Kry et al. 2002]



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We are about 100 times faster!



Conclusion

Linear Blend Skinning Decomposition Model

- Convex, sparse weights
- ✓ Rigid bone transformations
- ✓ Iterative bone transformation linear solvers
 - ✓ Nearly optimized, working well with highly deformation models
 - ✓ Fast
 - ✓ Simple



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Linear Blend Skinning Decomposition Model

- Convex, sparse weights
- ✓ Rigid bone transformations
- ✓ Iterative bone transformation linear solvers
 - ✓ Nearly optimized, working well with highly deformation models
 - ✓ Fast
 - ✓ Simple
- ★ Considering skeleton hierarchy
- ★ Utilizing other information: Mesh topology or anatomy



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http://graphics.cs.uh.edu/ble/papers/2012sa-ssdr/