SIGGRAPH ASIA 2012

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Smooth Skinning Decomposition with **Rigid** Bones

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UNIVERSITY of HOUSTON

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Motivation

Linear Blend Skinning (LBS)

- A.k.a. skeleton subspace deformation, enveloping, vertex blending, smooth skinning, bones skinning, etc.

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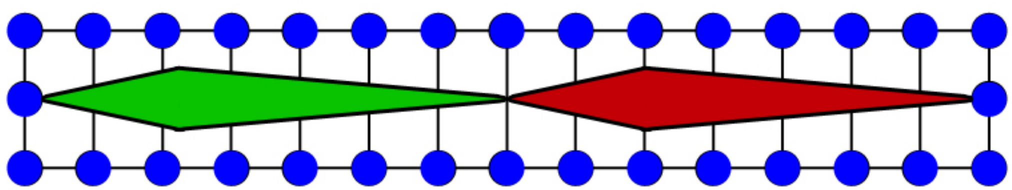

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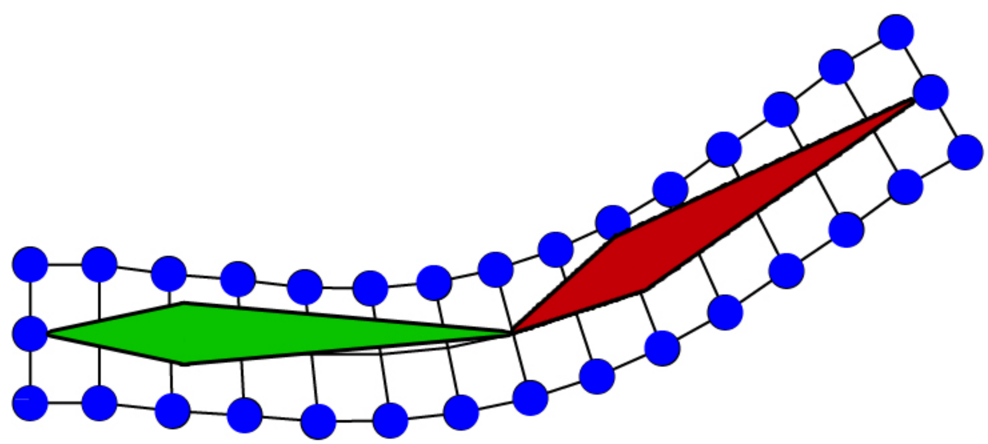

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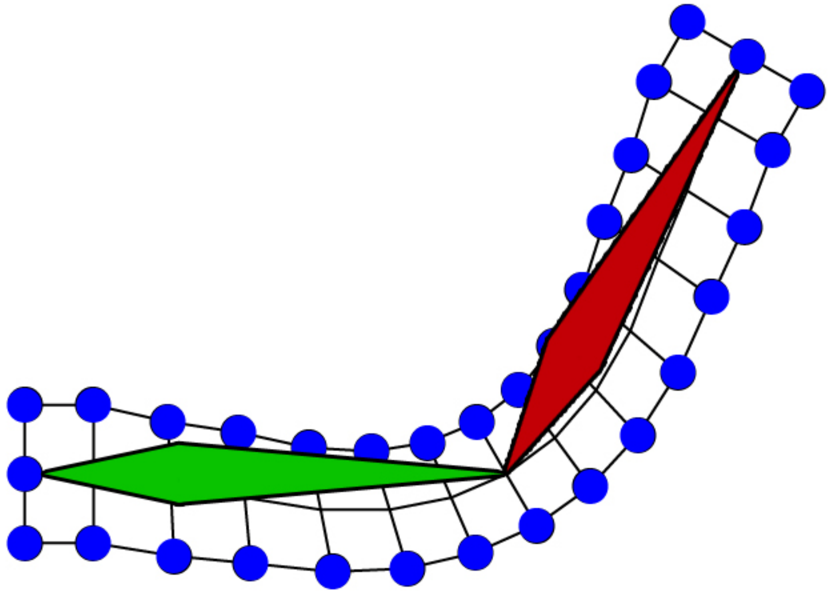

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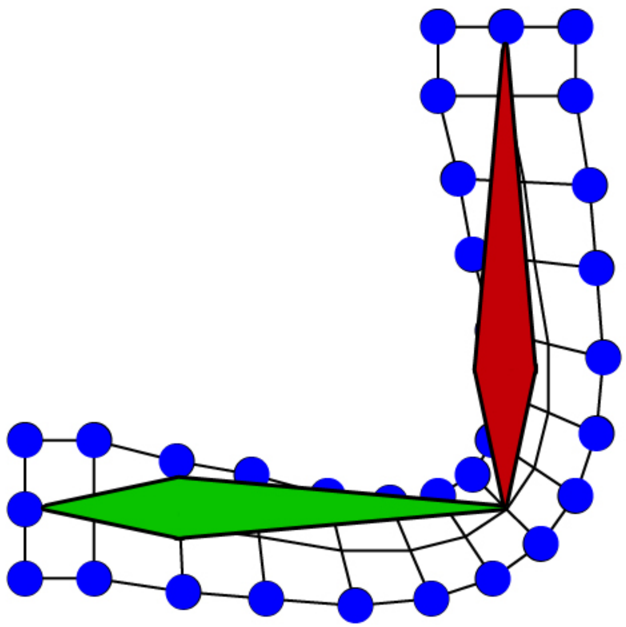

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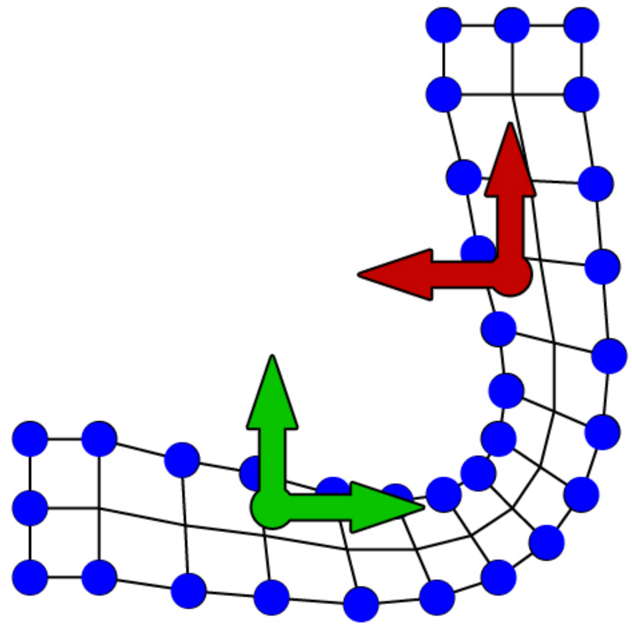

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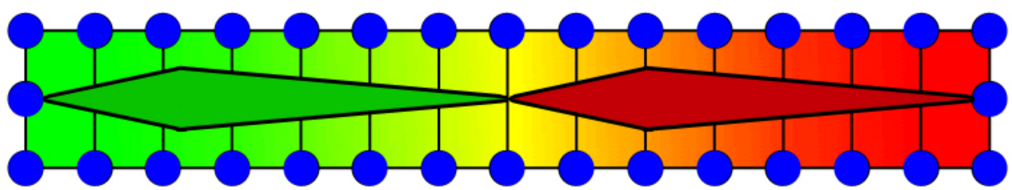

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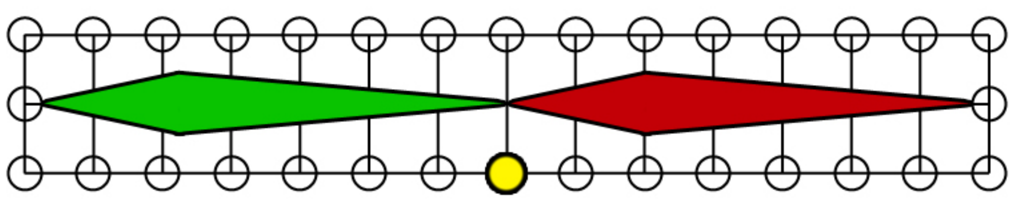

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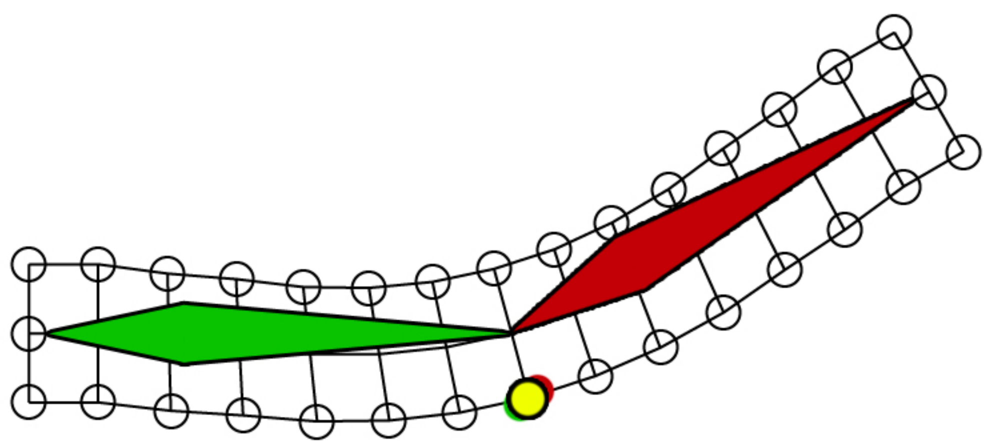

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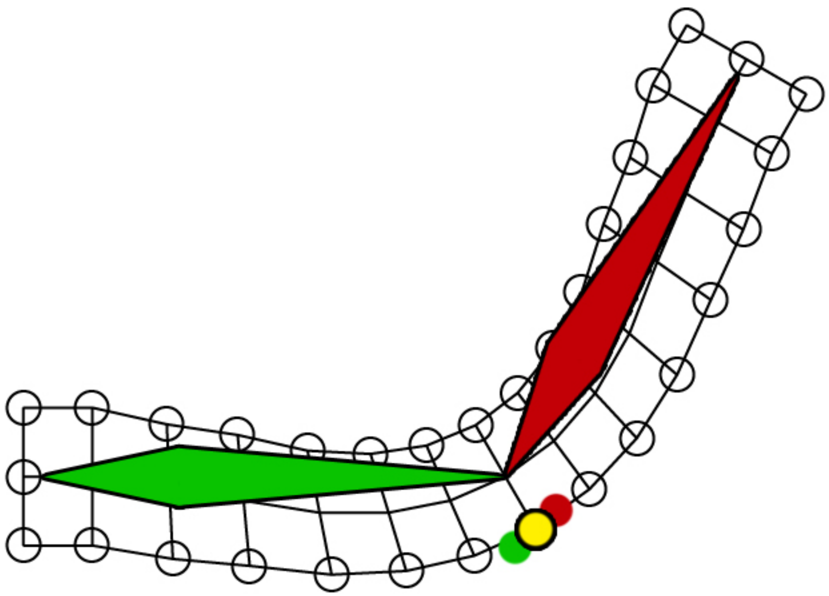

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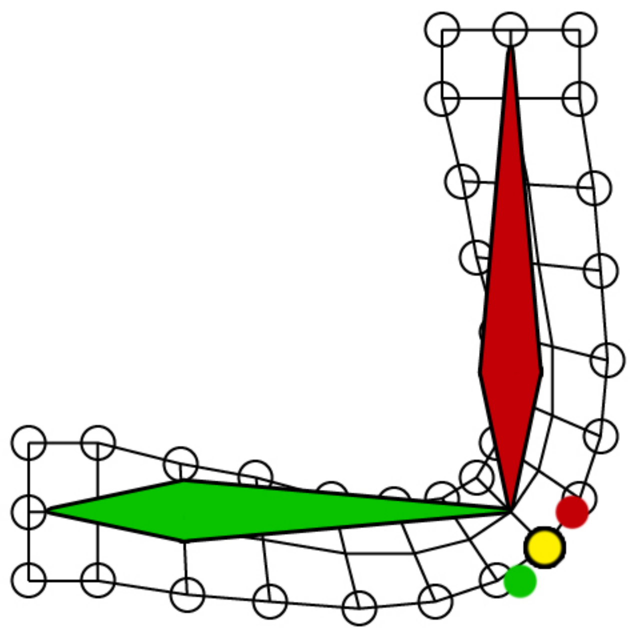

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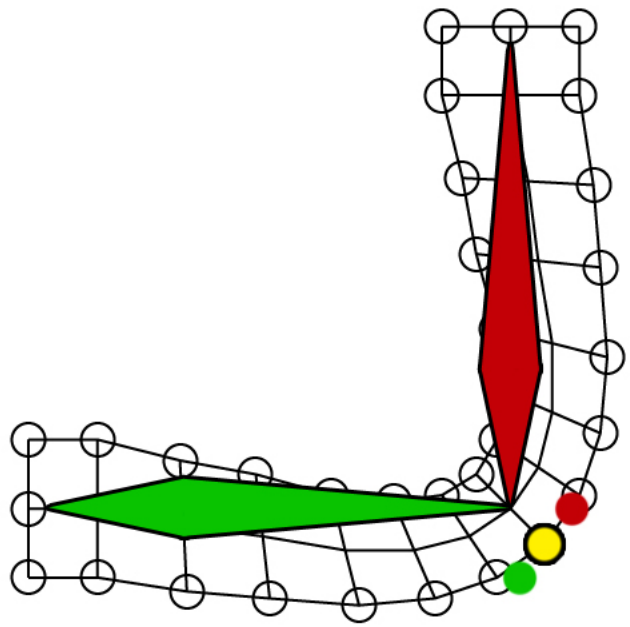


Number of bones

Rest pose

$$v_i = \sum_{j=1}^{|B|} w_{ij} (R_j p_i + T_j)$$

Weight

Bone rotation

Bone translation

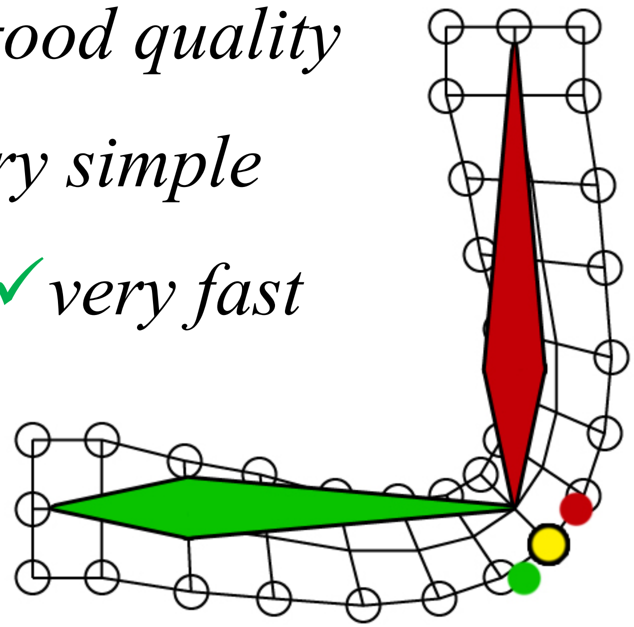
Linear Blend Skinning (LBS)

- A.k.a. skeleton subspace deformation, enveloping, vertex blending, smooth skinning, bones skinning, etc.

✓ *fairly good quality*

✓ *very simple*

✓ *very fast*



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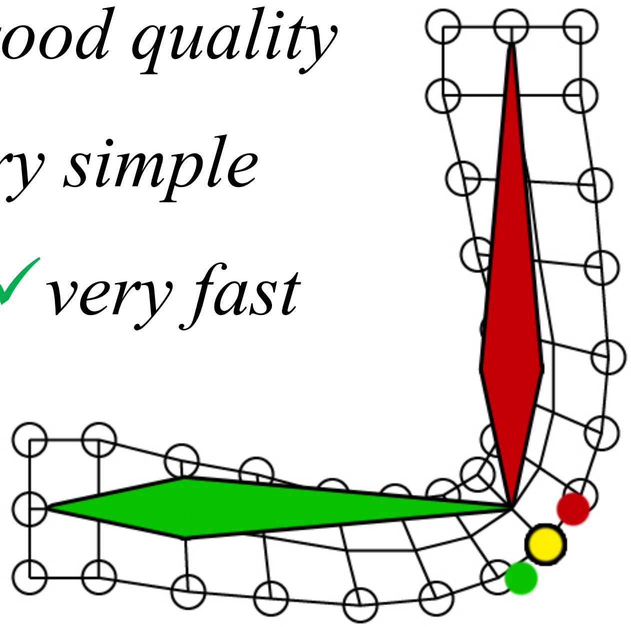
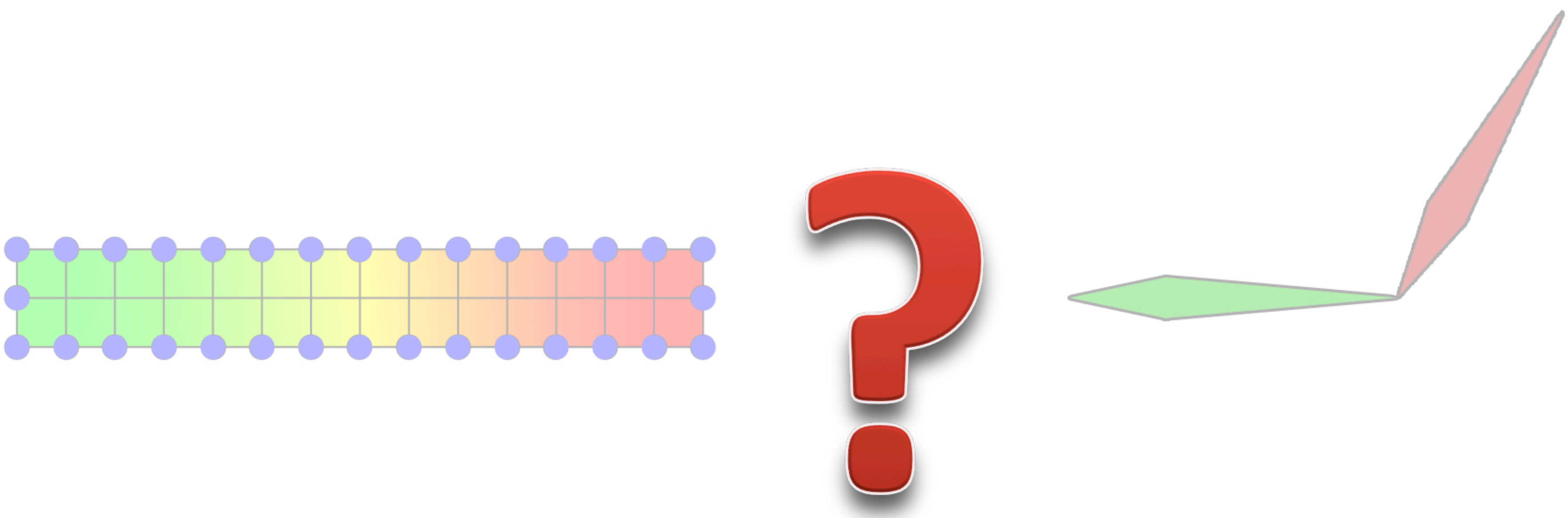


$$v_i = \sum_{j=1}^{|B|} w_{ij} (R_j p_i + T_j)$$

→ *most popular skinning model*

Linear Blend Skinning (LBS)



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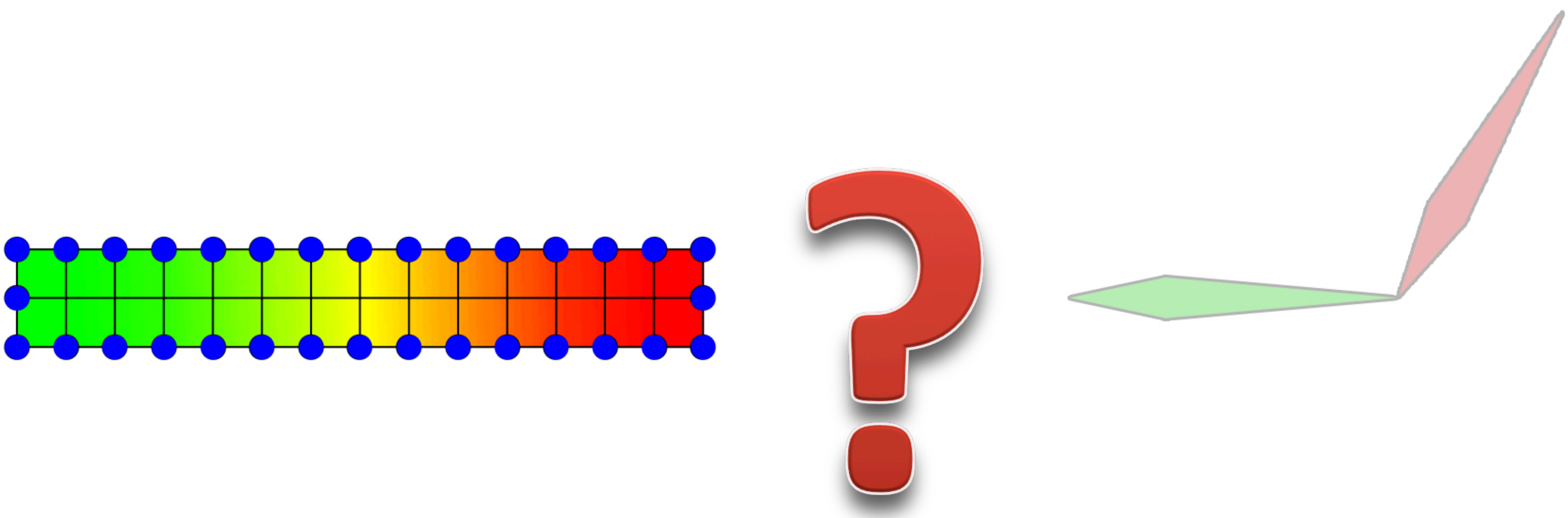

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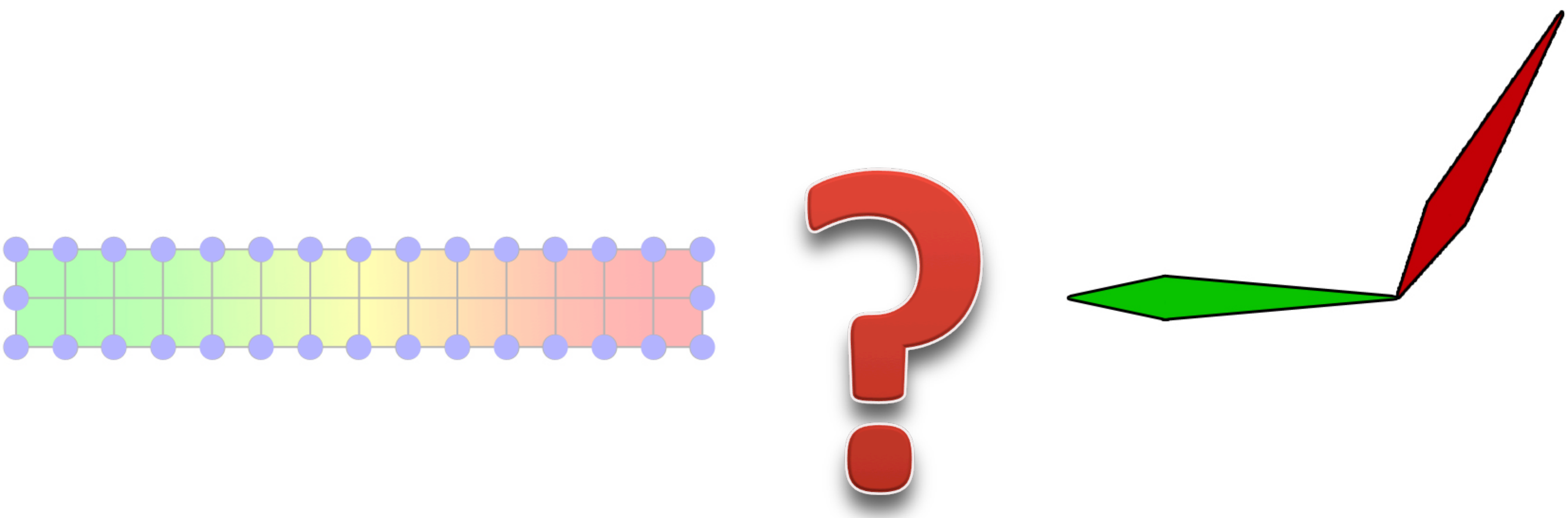
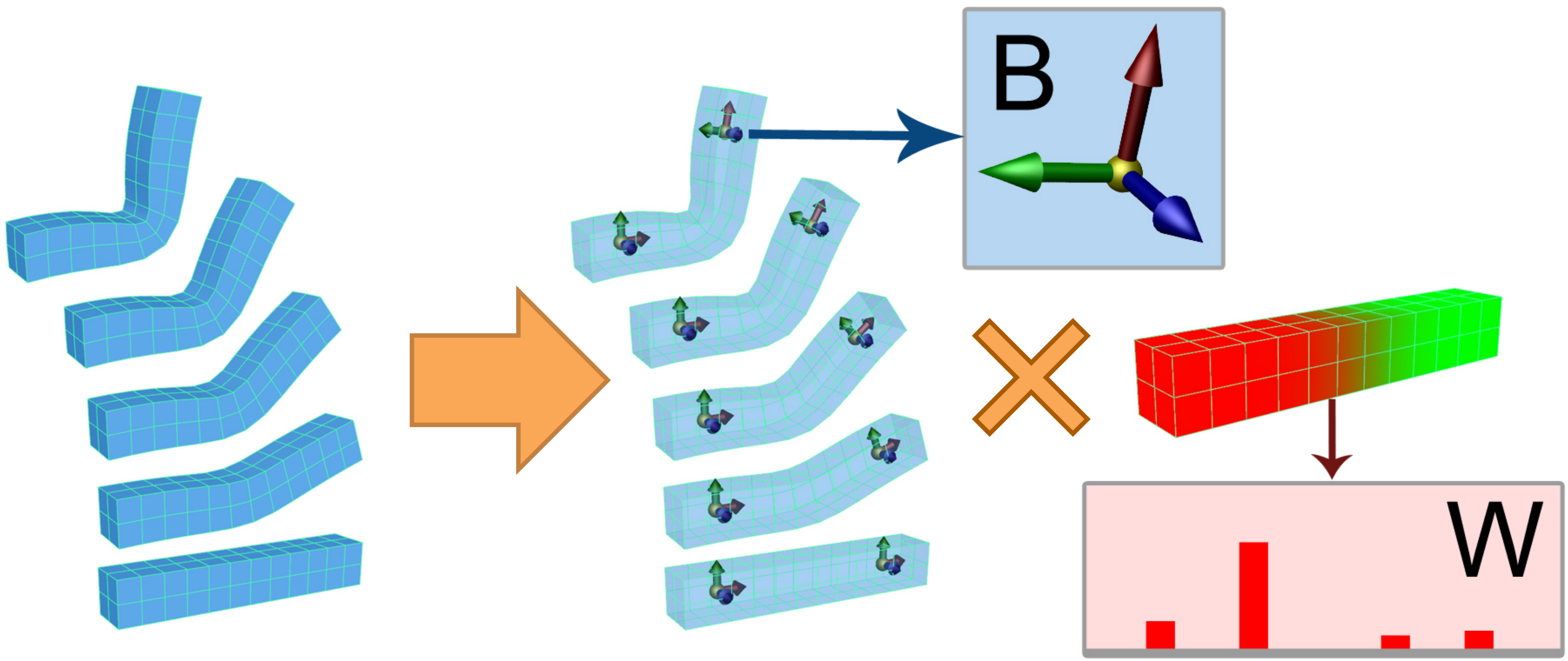


Skinning Decomposition

Skinning Decomposition



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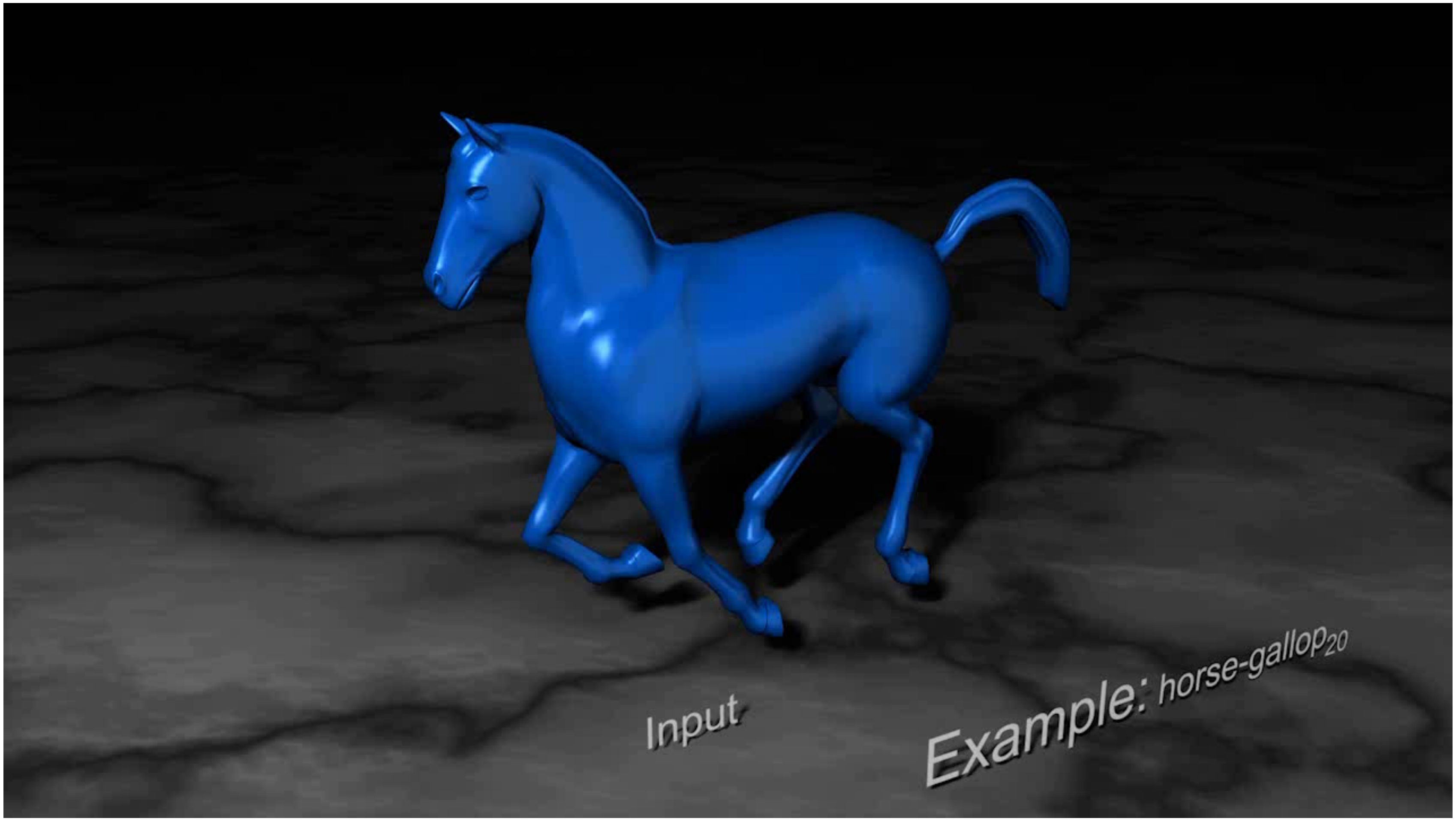
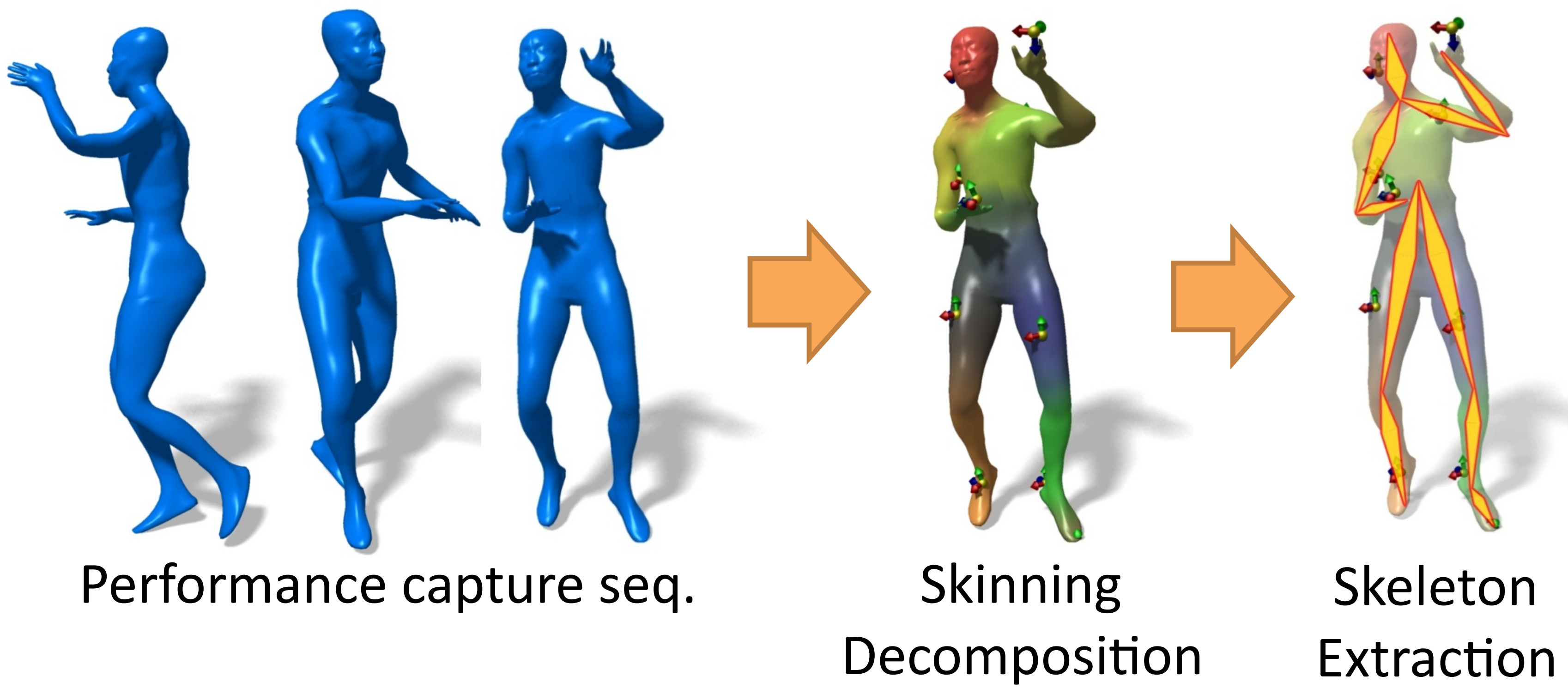


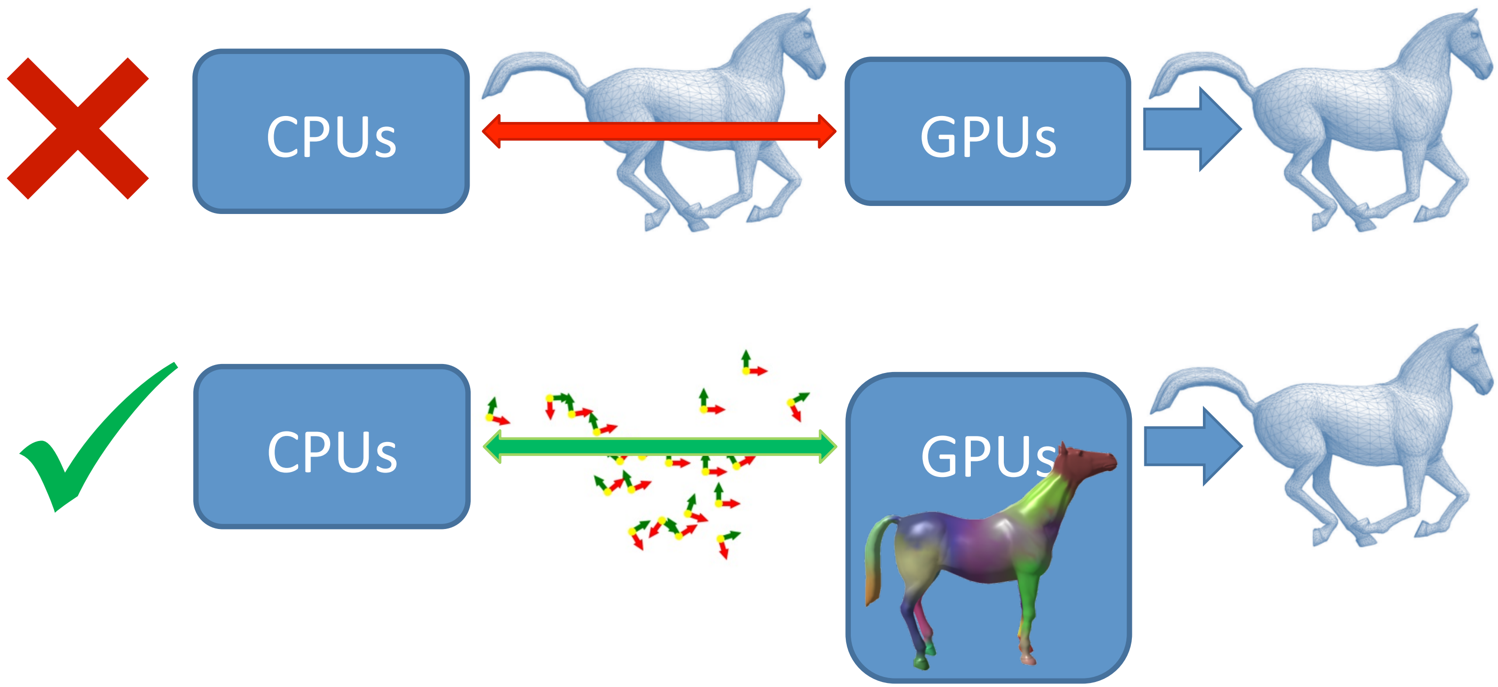
Applications

- Rigging, animation editing



Applications

- Rigging, animation editing
- Compression, hardware accelerated rendering



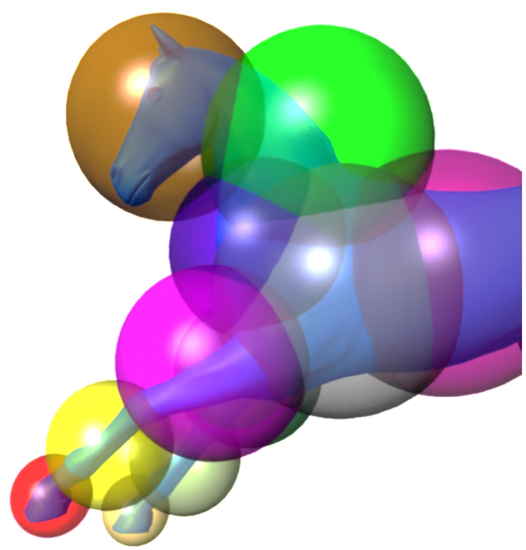
Applications

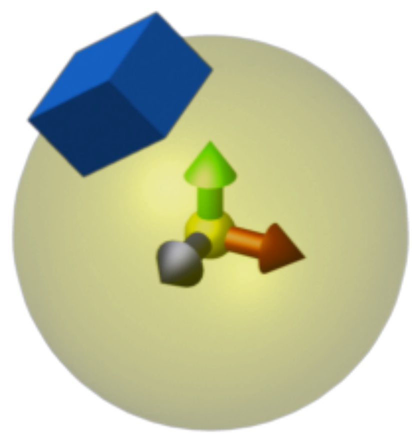
- Rigging, animation editing
- Compression, hardware accelerated rendering
- Segmentation, meshes simplification

Skinning Weights

Applications

- Rigging, animation editing
- Compression, hardware accelerated rendering
- Segmentation, meshes simplification
- Collision detection





Applications

- Rigging, animation editing
- Compression, hardware accelerated rendering
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- Collision detection

Skinning Decomposition

Smooth Skinning Decomposition with Rigid Bones

Input: Example poses

Output: Linear Blend Skinning model

- Sparse, convex weights
- **Rigid bone transformations**
- No skeleton hierarchy

Smooth Skinning Decomposition with Rigid Bones

Input: Example poses

Output: Linear Blend Skinning model

- Sparse, convex weights
- **Rigid bone transformations**
- No skeleton hierarchy

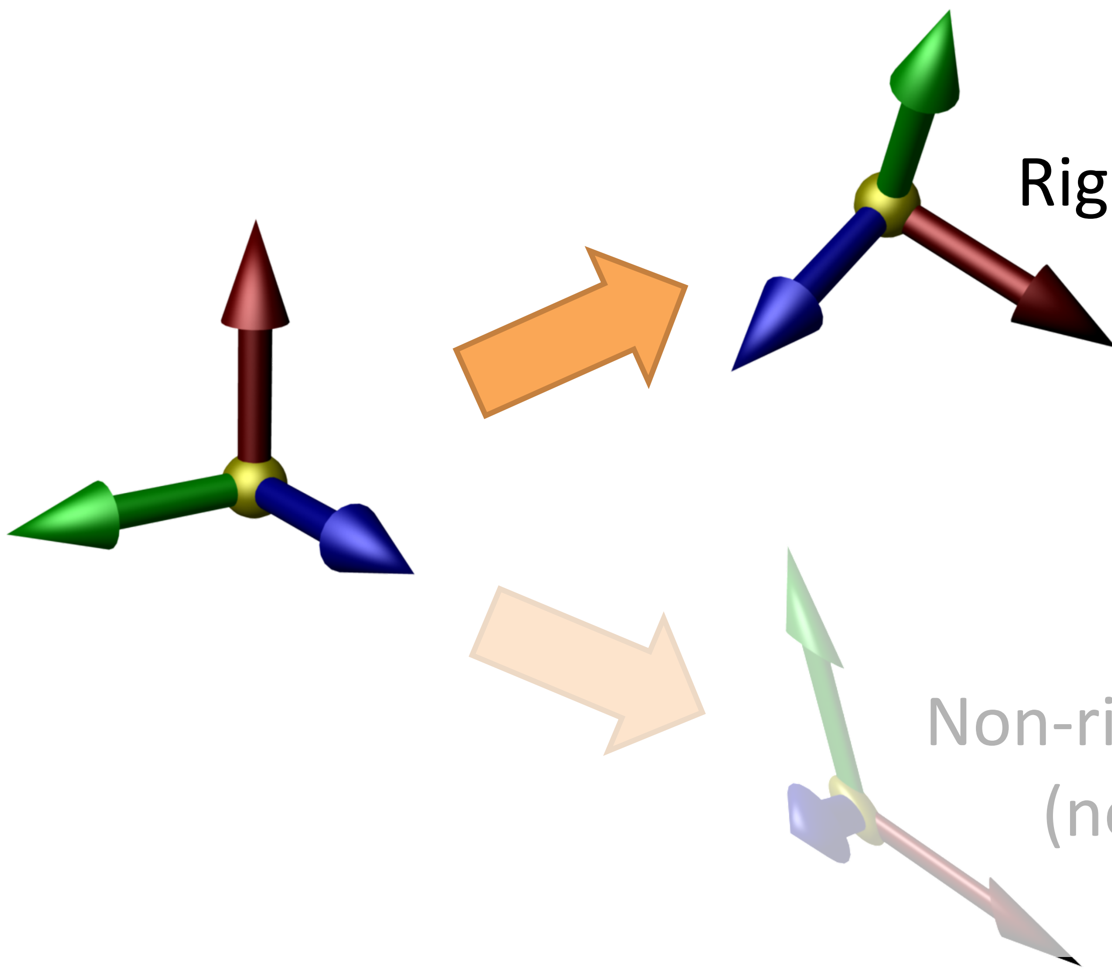
Goals:

✓ Approximate highly deformation models

✓ Fast performance

✓ Simple implementation

Rigid Bones v.s. Flexible Bones



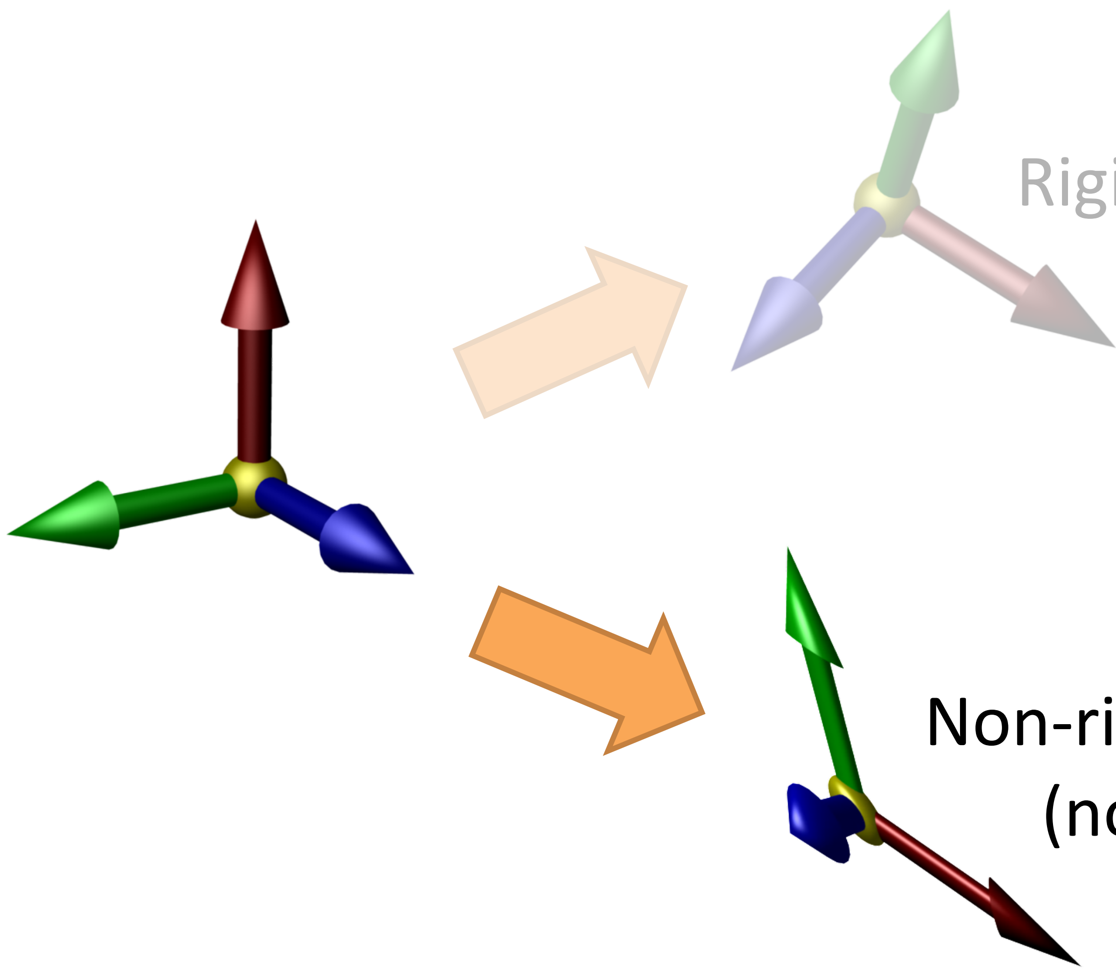
Rigid transformation
(orthogonal)

$$\begin{bmatrix} r_{11} & r_{12} & r_{13} & t_1 \\ r_{21} & r_{22} & r_{23} & t_2 \\ r_{31} & r_{32} & r_{33} & t_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$R^{\mathrm{T}}R = I, \det R = 1$$

Non-rigid transformation
(non-orthogonal)

Rigid Bones v.s. Flexible Bones



$$\begin{bmatrix} r_{11} & r_{12} & r_{13} & t_1 \\ r_{21} & r_{22} & r_{23} & t_2 \\ r_{31} & r_{32} & r_{33} & t_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

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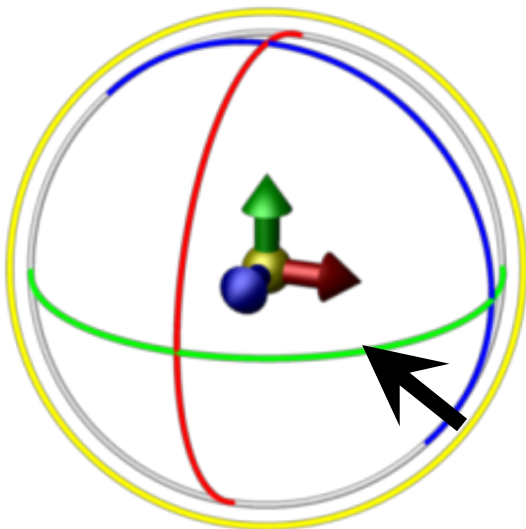
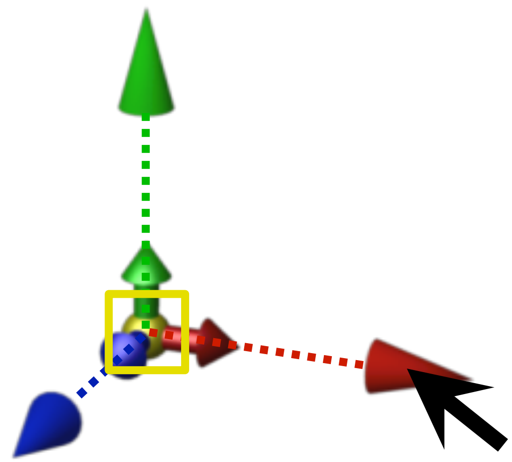
Benefits of Rigid Bones

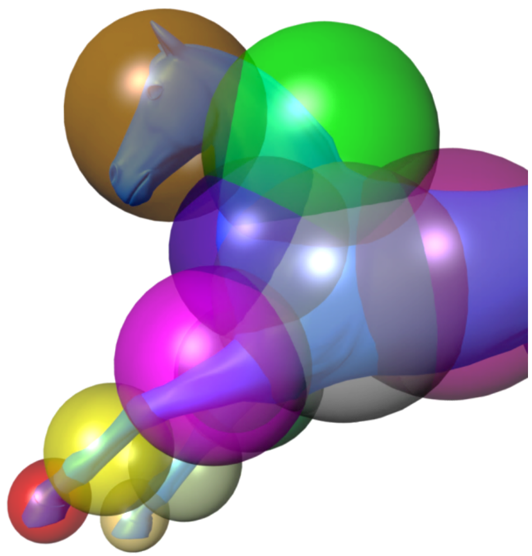
- ✓ Animation editing

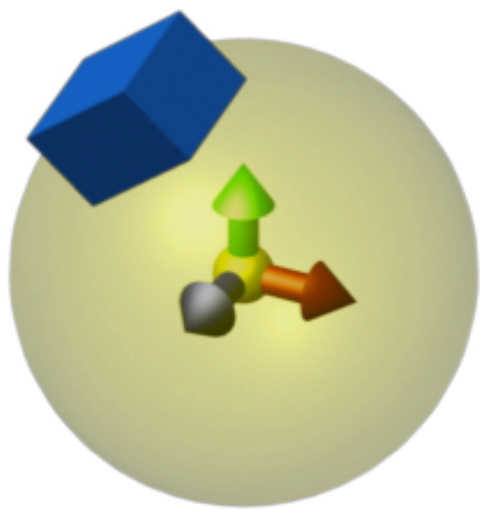




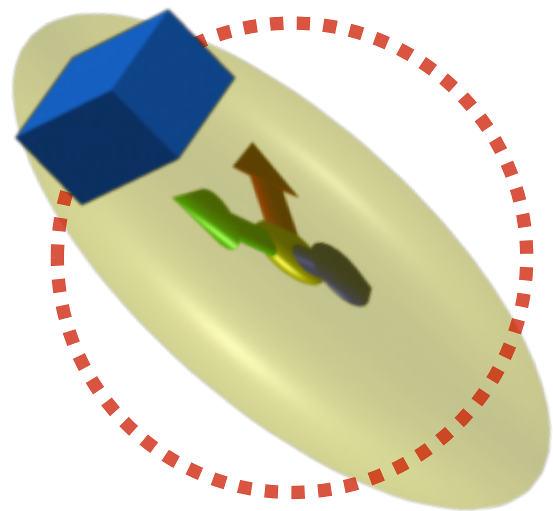
Benefits of Rigid Bones

- ✓ Animation editing
- ✓ Collision detection





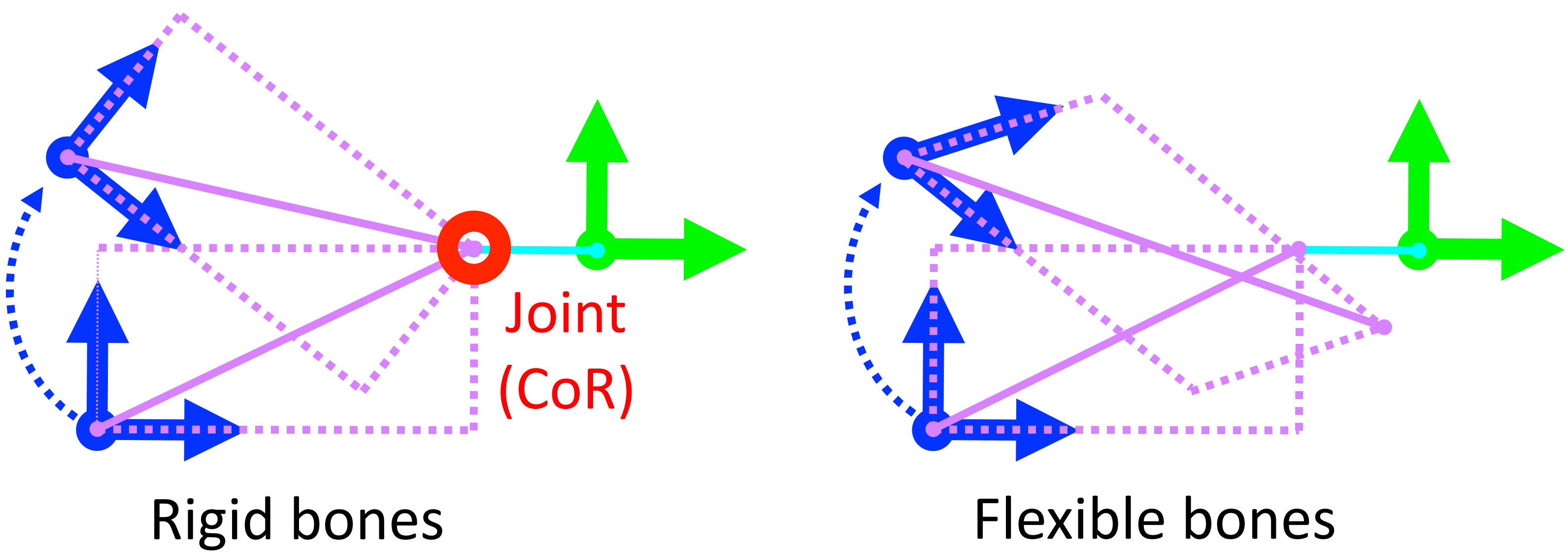
Rigid bone



Flexible bone

Benefits of Rigid Bones

- ✓ Animation editing
- ✓ Collision detection
- ✓ Skeleton extraction



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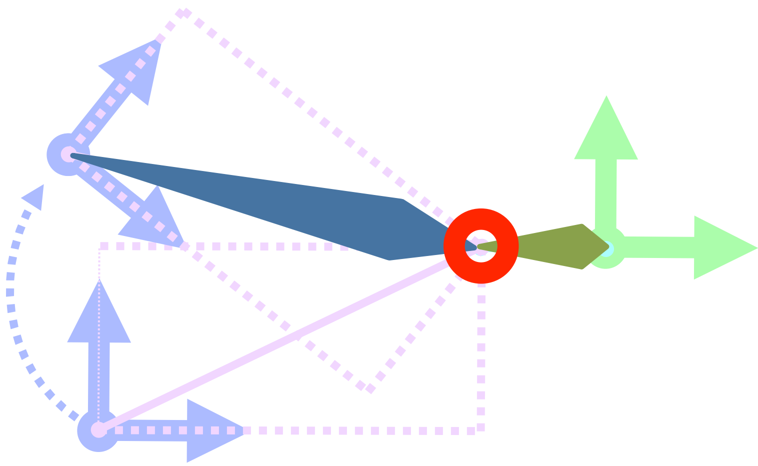
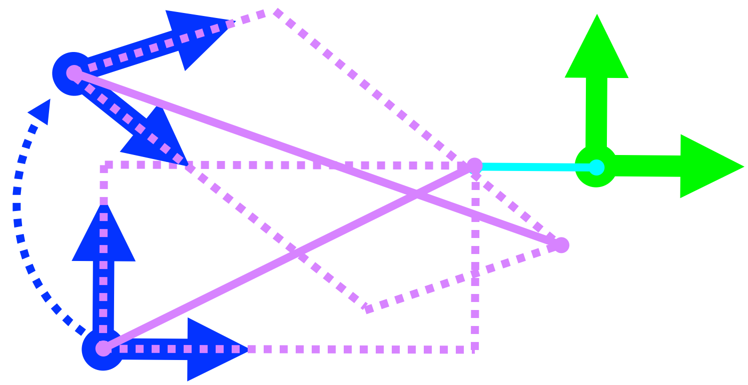


Rigid bones



Flexible bones

Benefits of Rigid Bones

- ✓ Animation editing
- ✓ Collision detection
- ✓ Skeleton extraction
- ✓ Compact representation

$$(r_1, r_2, r_3, t_1, t_2, t_3) \qquad \text{v.s.} \qquad \begin{bmatrix} r_{11} & r_{12} & r_{13} & t_1 \\ r_{21} & r_{22} & r_{23} & t_2 \\ r_{31} & r_{32} & r_{33} & t_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Rigid bone
6 DOFs

Flexible bone
12 DOFs

Benefits of Rigid Bones

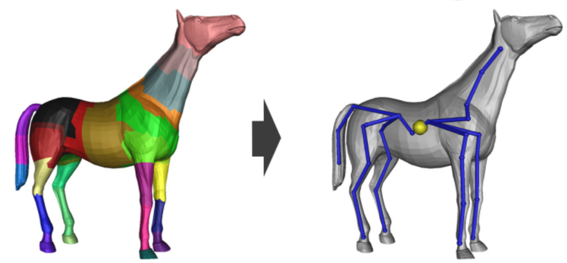
- ✓ Animation editing
- ✓ Collision detection
- ✓ Skeleton extraction
- ✓ Compact representation

Rigid Bones

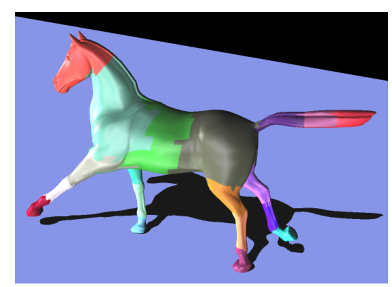
Previous Work

- Cluster triangles with similar deformations to get bones, then optimize skinning weights
  - Skinning Mesh Animations [James and Twigg 2005]

  - Example-Based Skeleton Extraction [Schaefer and Yuksel 2007]



  - Automatic Conversion of Mesh Animations into Skeleton-based Animations [de Aguiar et al. 2008]



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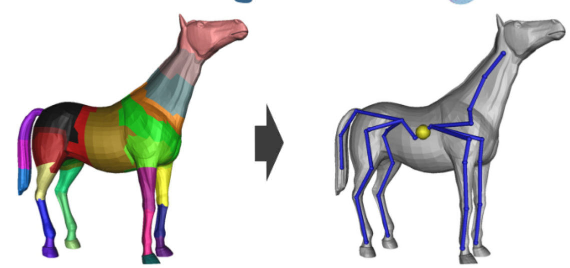

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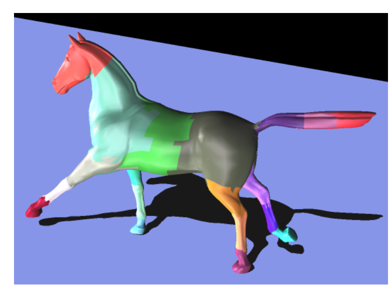
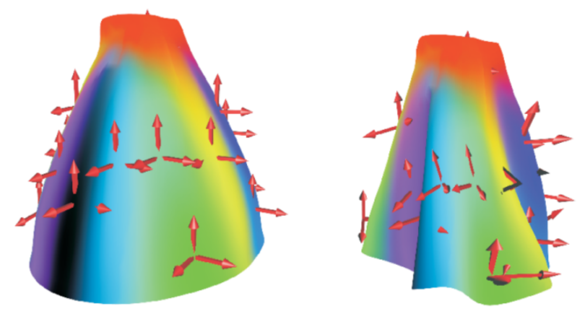


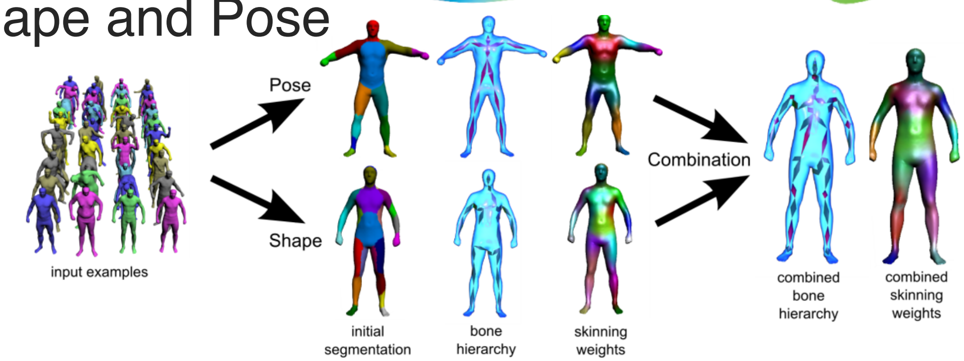
❌ Not consider skin blending,
only good for nearly articulated models

Previous Work

- Joint optimize bone transformations and skinning weights
  - Fast and Efficient Skinning of Animated Meshes [Kavan et al. 2010]
    - ✓ Linear solvers
    - ✗ Flexible bones



  - Learning Skeletons for Shape and Pose [Hasler et al. 2010]
    - ✓ Rigid bones
    - ✗ Non linear solver



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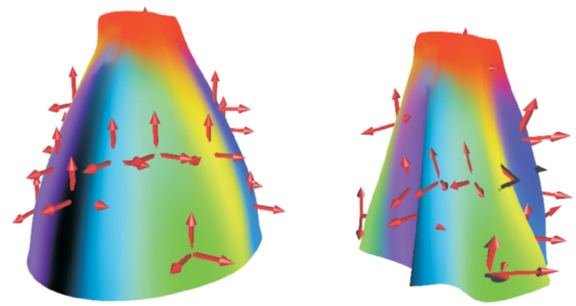

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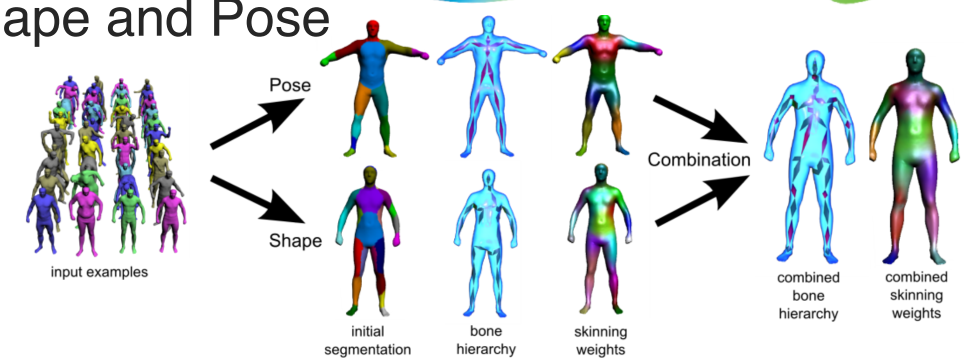

✓ Good approximation of highly deformation models

! Non-convex optimization, possibly with non-linear constraints

Smooth Skinning Decomposition with Rigid Bones

[Le and Deng 2012]

- ✓ Rigid bones
- ✓ Highly deformation models
- ✓ Linear solvers

Problem Formulation

$$\min_{w,R,T} E = \min_{w,R,T} \sum_{t=1}^{|t|} \sum_{i=1}^{|V|} \left\| v_i^t - \sum_{j=1}^{|B|} w_{ij} (R_j^t p_i + T_j^t) \right\|^2$$

Subject to: $w_{ij} \geq 0, \forall i, j$

$$\sum_{j=1}^{|B|} w_{ij} = 1, \forall i$$

$$|\{w_{ij} | w_{ij} \neq 0\}| \leq |K|, \forall i$$

$$R_j^{t\,\mathsf{T}} R_j^t = I, \det R_j^t = 1, \forall t, j$$

Problem Formulation

$$\min_{w,R,T} E = \min_{w,R,T} \sum_{t=1}^{|t|} \sum_{i=1}^{|V|} \left\| v_i^t - \sum_{j=1}^{|B|} w_{ij} (R_j^t p_i + T_j^t) \right\|^2$$

Approximation Error

Subject to: $w_{ij} \geq 0, \forall i, j$

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$$\min_{w,R,T} E = \min_{w,R,T} \sum_{t=1}^{|t|} \sum_{i=1}^{|V|} \left\| v_i^t - \sum_{j=1}^{|B|} w_{ij} (R_j^t p_i + T_j^t) \right\|^2$$

Skinning Weights

Subject to: $w_{ij} \geq 0, \forall i, j$

$$\sum_{j=1}^{|B|} w_{ij} = 1, \forall i$$

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$${R_j^t}^{\mathsf{T}} R_j^t = I, \det R_j^t = 1, \forall t, j$$

Problem Formulation

$$\min_{w,R,T} E = \min_{w,R,T} \sum_{t=1}^{|t|} \sum_{i=1}^{|V|} \left\| v_i^t - \sum_{j=1}^{|B|} w_{ij} \left(\boxed{R_j^t} p_i + \boxed{T_j^t} \right) \right\|^2$$

Subject to: $w_{ij} \geq 0, \forall i, j$

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Bone Transformations

Problem Formulation

$$\min_{w,R,T} E = \min_{w,R,T} \sum_{t=1}^{|t|} \sum_{i=1}^{|V|} \left\| v_i^t - \sum_{j=1}^{|B|} w_{ij} (R_j^t p_i + T_j^t) \right\|^2$$

Subject to: $w_{ij} \geq 0, \forall i, j$ → Non-negativity

$$\sum_{j=1}^{|B|} w_{ij} = 1, \forall i$$

$$|\{w_{ij} | w_{ij} \neq 0\}| \leq |K|, \forall i$$

$${R_j^t}^{\mathsf{T}} R_j^t = I, \det R_j^t = 1, \forall t, j$$

Problem Formulation

$$\min_{w,R,T} E = \min_{w,R,T} \sum_{t=1}^{|t|} \sum_{i=1}^{|V|} \left\| v_i^t - \sum_{j=1}^{|B|} w_{ij} (R_j^t p_i + T_j^t) \right\|^2$$

Subject to: $w_{ij} \geq 0, \forall i, j$

$$\sum_{j=1}^{|B|} w_{ij} = 1, \forall i$$ → Affinity

$$|\{w_{ij} | w_{ij} \neq 0\}| \leq |K|, \forall i$$

$${R_j^t}^{\mathsf{T}} R_j^t = I, \det R_j^t = 1, \forall t, j$$

Problem Formulation

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Subject to: $w_{ij} \geq 0, \forall i, j$

$$\sum_{j=1}^{|B|} w_{ij} = 1, \forall i$$

$$|\{w_{ij} | w_{ij} \neq 0\}| \leq |K|, \forall i$$ → Sparseness

$${R_j^t}^{\mathsf{T}} R_j^t = I, \det R_j^t = 1, \forall t, j$$

Problem Formulation

$$\min_{w,R,T} E = \min_{w,R,T} \sum_{t=1}^{|t|} \sum_{i=1}^{|V|} \left\| v_i^t - \sum_{j=1}^{|B|} w_{ij} (R_j^t p_i + T_j^t) \right\|^2$$

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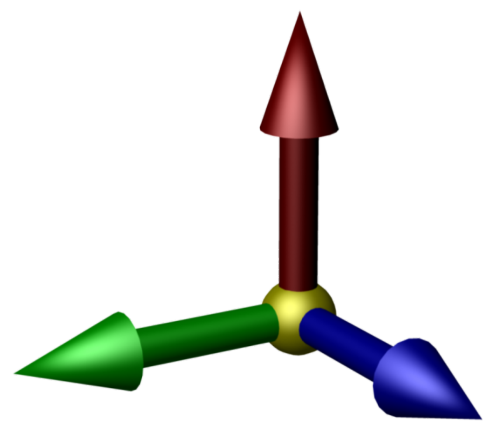
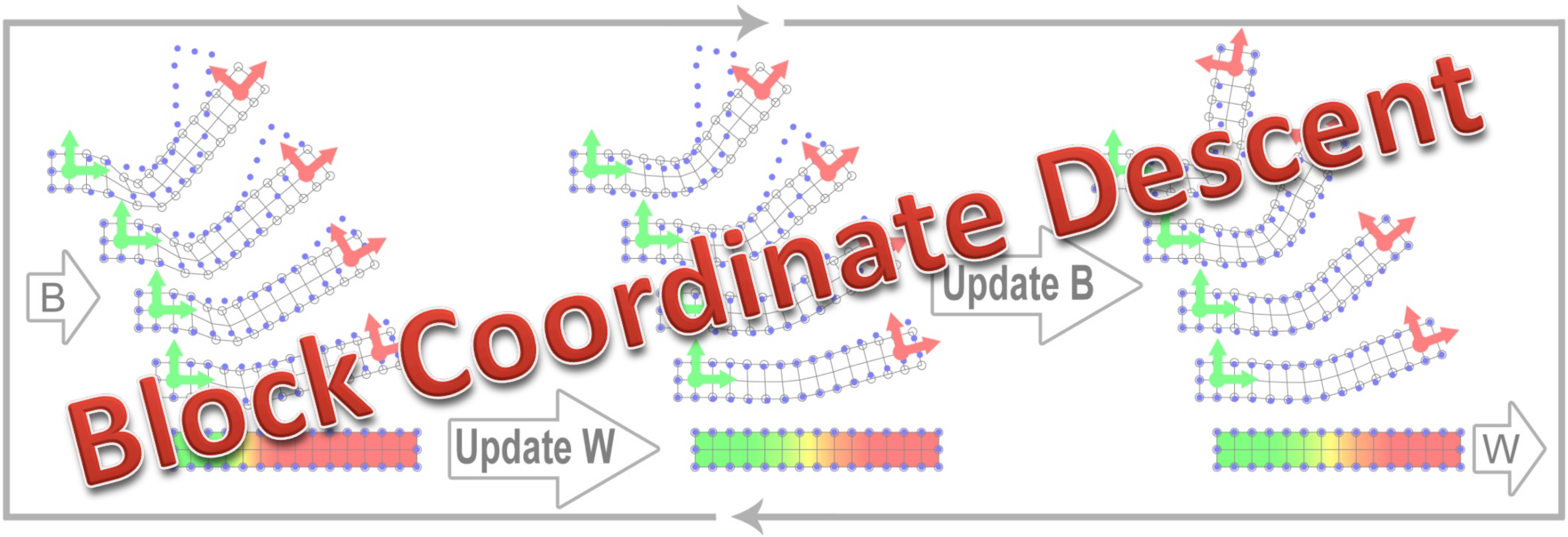
→ Orthogonal



! Non-linear

Skinning Decomposition Algorithm

$$\min_{w,R,T} E = \min_{w,R,T} \sum_{t=1}^{|t|} \sum_{i=1}^{|V|} \left\| v_i^t - \sum_{j=1}^{|B|} w_{ij} (R_j^t p_i + T_j^t) \right\|^2$$



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Skinning Decomposition Algorithm

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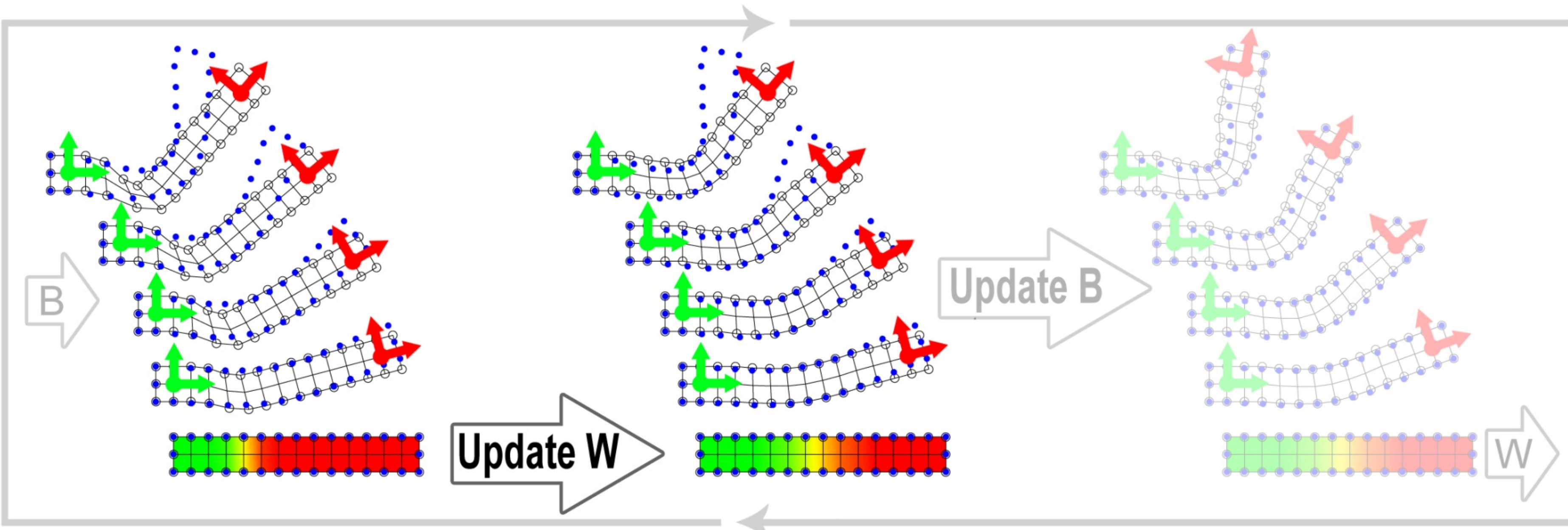

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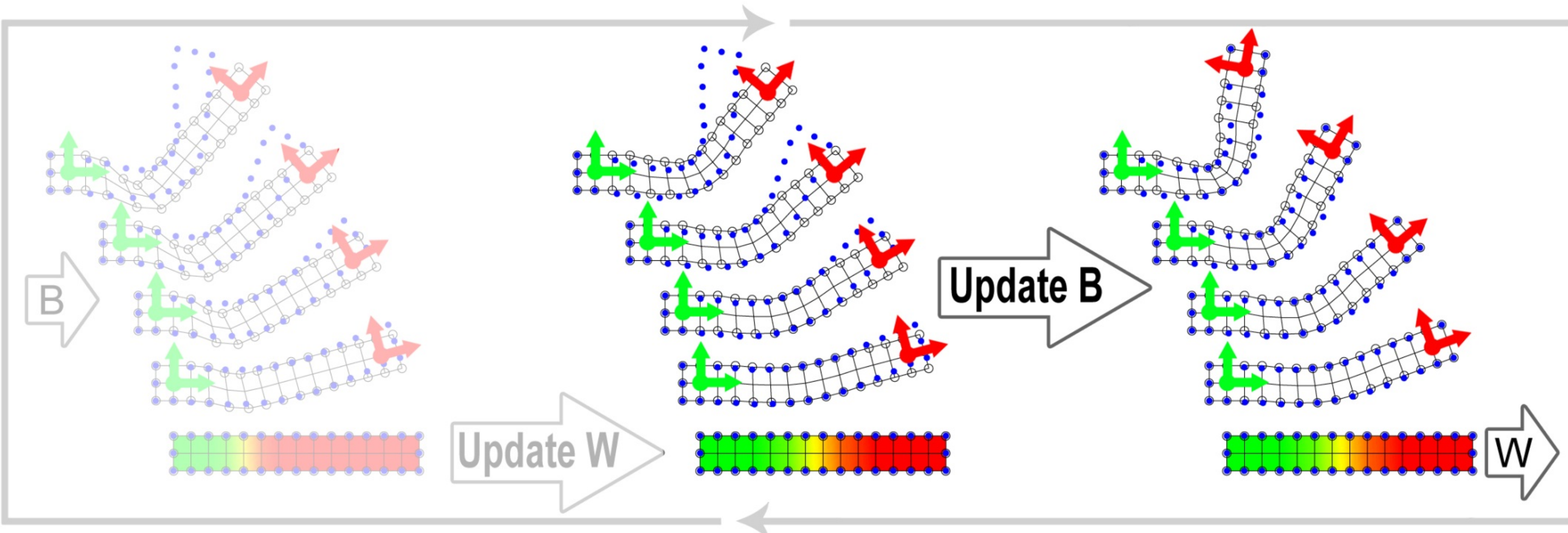

Initialization

- No blending (rigid binding): each vertex is driven by exactly one bone

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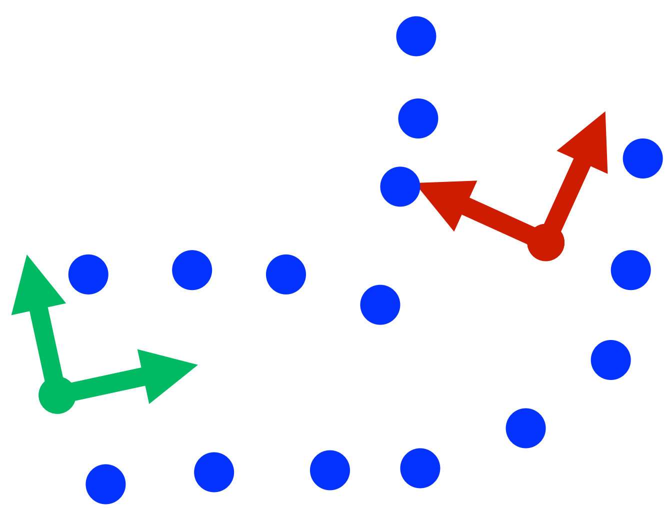

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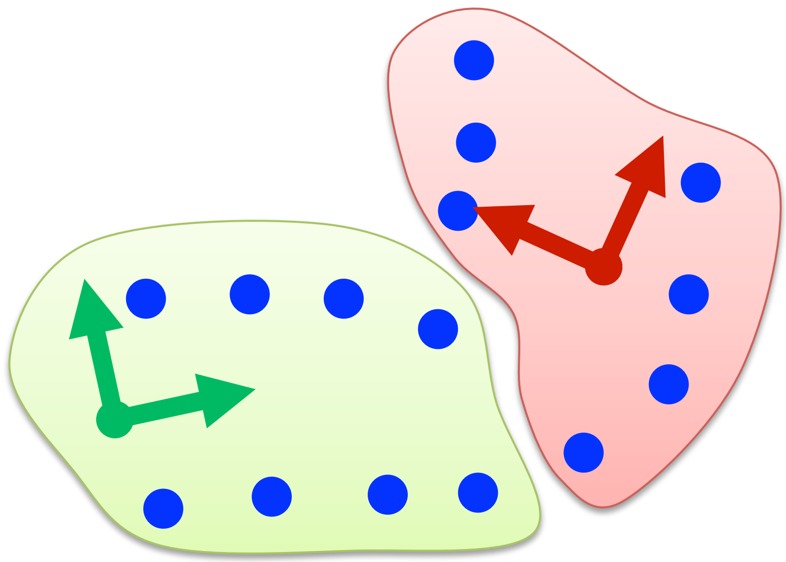

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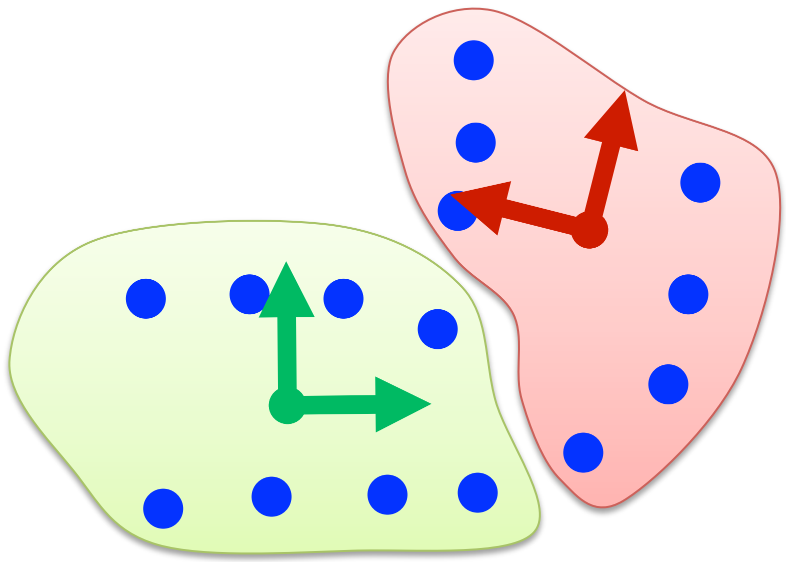

Skinning Weights Solver

- Per vertex solver: Constrained Linear Least Squares

$$W_i^{\mathsf{T}} = \arg\min_x \|Ax - b\|^2$$

Subject to: $x \geq 0$

$$\|x\|_1 = 1$$

$$\|x\|_0 \leq |K|$$

Skinning Weights Solver

- Per vertex solver: Constrained Linear Least Squares

$$W_i^{\mathsf{T}} = \arg\min_x \|Ax - b\|^2$$

Subject to: $x \geq 0$ → Bound Constraint

$\|x\|_1 = 1$ → Equality Constraint

$\|x\|_0 \leq |K|$

- Active Set Method [Lawson and Hanson]
  - Pre-compute LU factorization of $A^{\mathsf{T}}A$ and $A^{\mathsf{T}}b$
  - Pre-compute QR decomposition of $\begin{bmatrix}1 & 1 & \cdots & 1\end{bmatrix}^{\mathsf{T}}$

Skinning Weights Solver

- Per vertex solver: Constrained Linear Least Squares

$$W_i^{\mathsf{T}} = \arg\min_x \|Ax - b\|^2$$

Subject to: $x \geq 0$

$$\|x\|_1 = 1$$

$$\|x\|_0 \leq |K|$$ → Sparseness Constraint

- Weight pruning of bones with small contribution

$$e_{ij} = \left\| w_{ij}(R_j^t p_i + T_j^t) \right\|^2$$

Keep $|K|$ bones with largest e_{ij} and solve the LS again

Bone Transformations Solver

- Per example pose solver:

$$\min_{R^t,T^t} E^t = \min_{R^t,T^t} \sum_{i=1}^{|V|} \left\| v_i^t - \sum_{j=1}^{|B|} w_{ij}(R_j^t p_i + T_j^t) \right\|^2$$

$$\text{Subject to: } \boxed{{R_j^t}^\top R_j^t = I,} \det R_j^t = 1$$

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? Levenberg-Marquardt optimization

✓ Optimized solution ✗ Slow

? Absolute Orientation (a.k.a. Procrustes Analysis) [Kabsch 1978; Horn 1987]

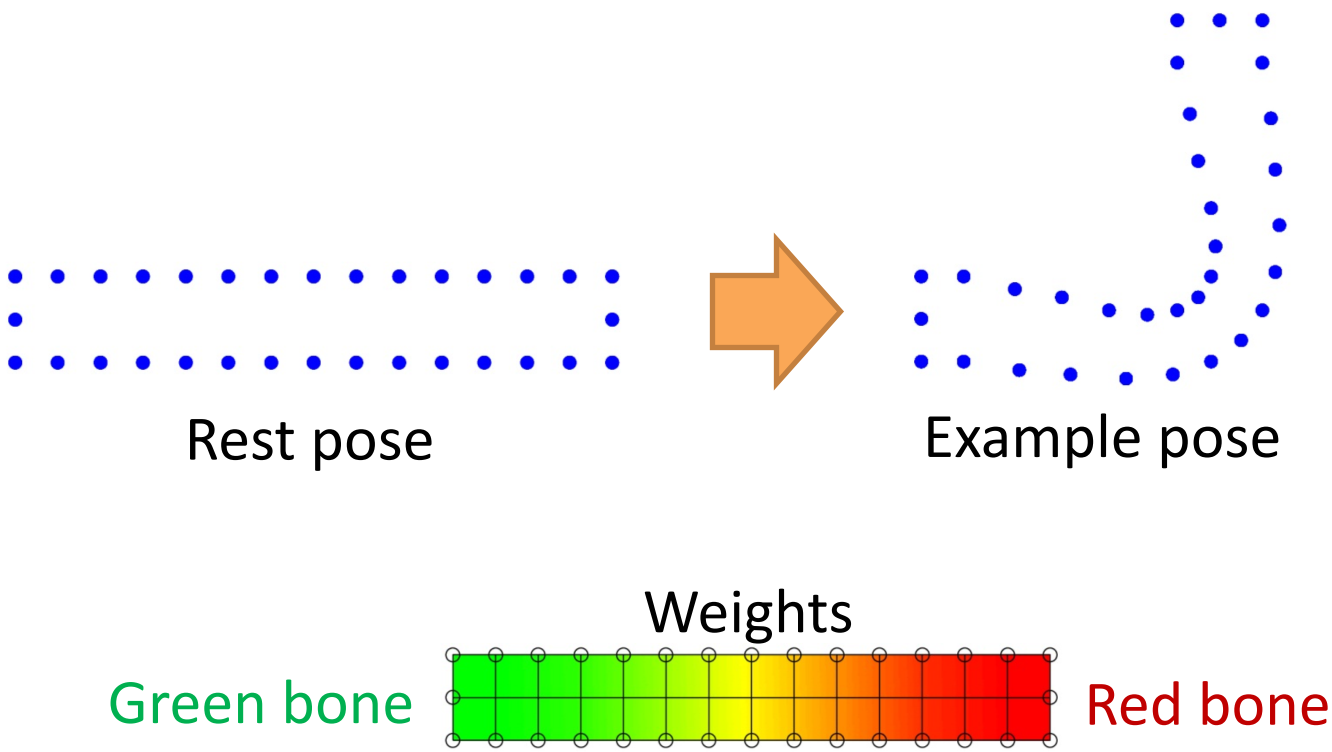
✓ Fast ✗ Approximate solution

Bone Transformations Solver

- Our solution: Solve bone transformation **one-by-one** to minimize the deformation residual of remaining bones
  - ✓ Linear solver, fast, and simple
  - ✓ Near optimized solution

Bone Transformations Solver

- Our solution: Solve bone transformation **one-by-one** to minimize the deformation residual of remaining bones



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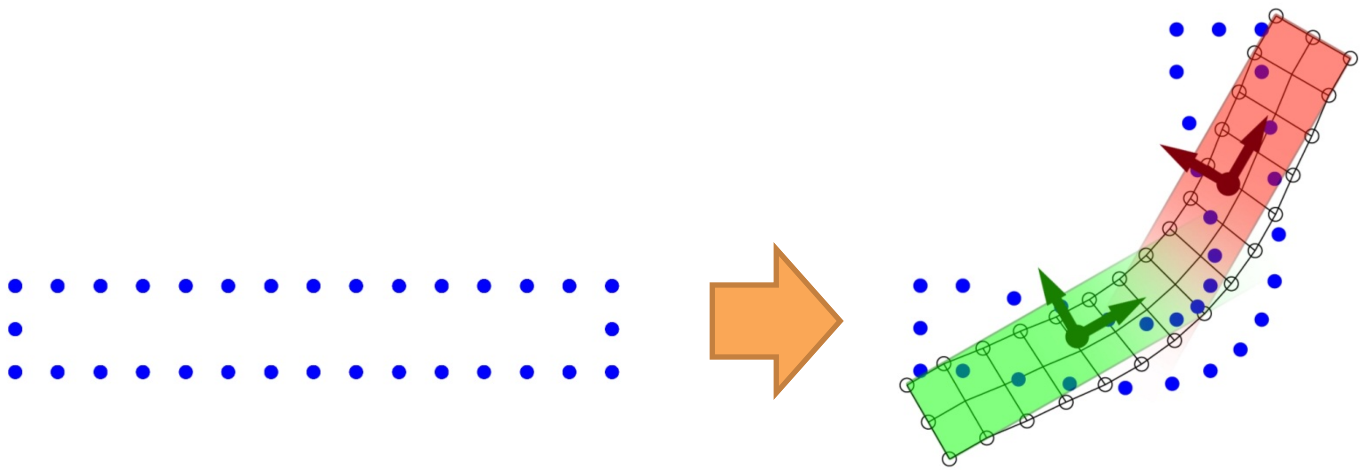

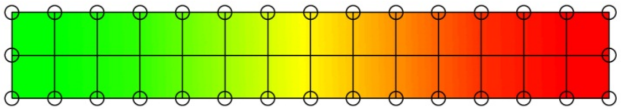

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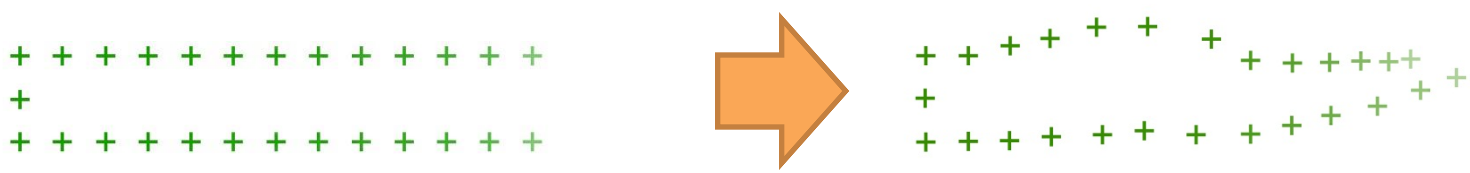

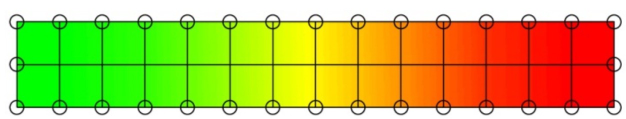

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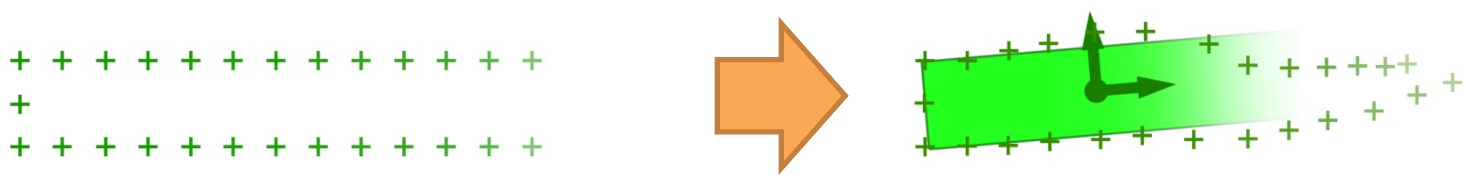

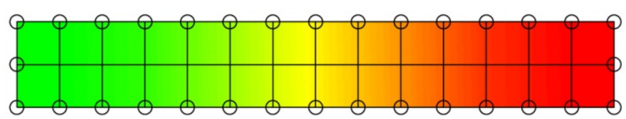

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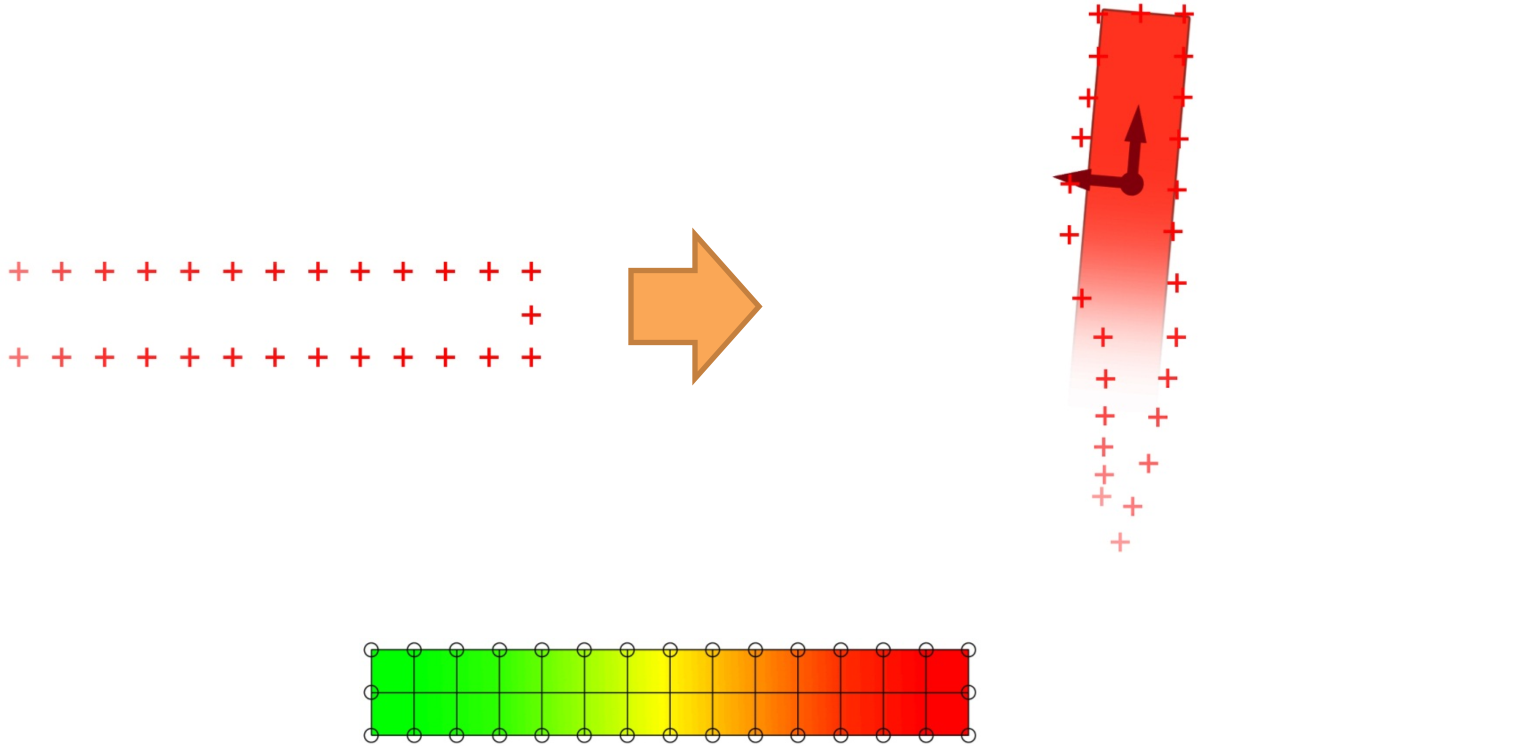

Bone Transformations Solver

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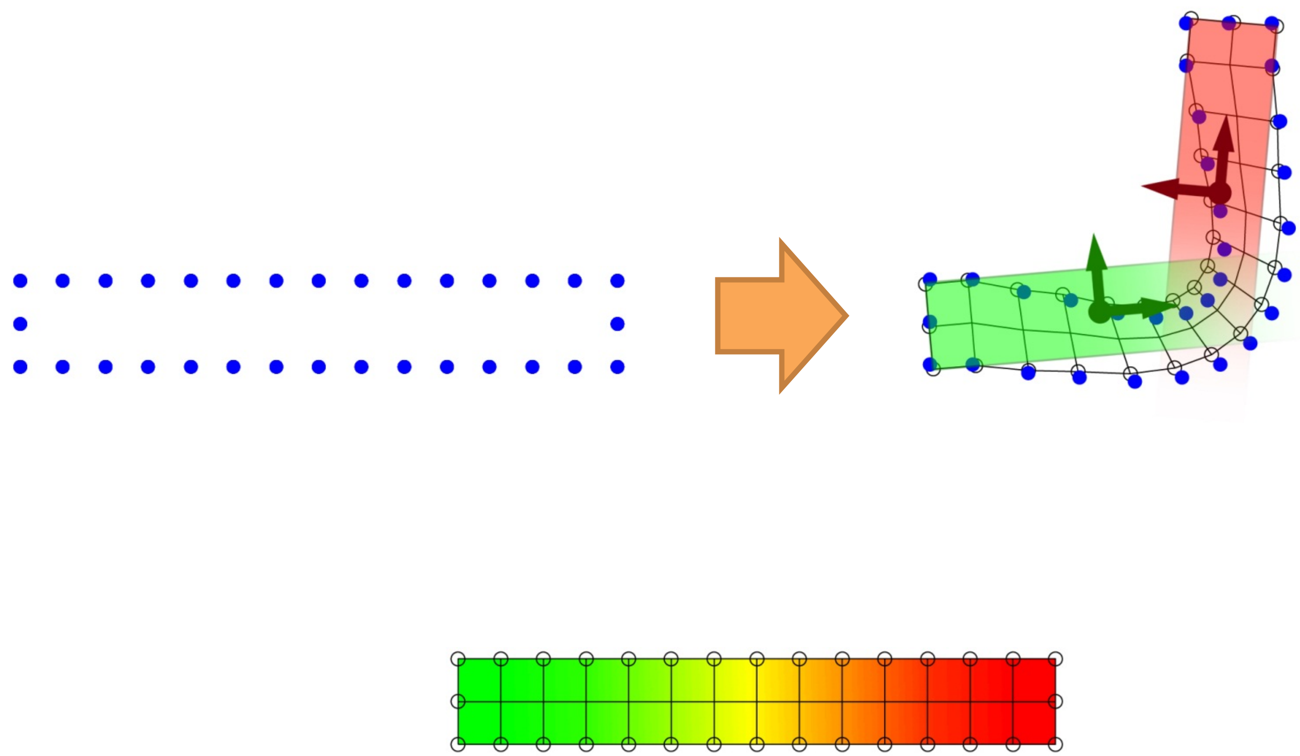

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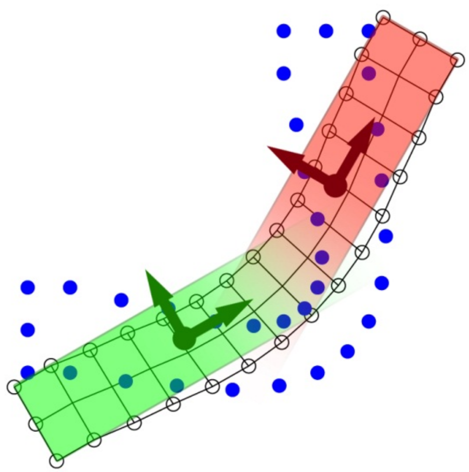
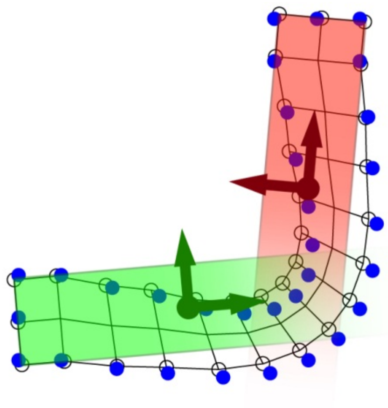


Before



After

Bone Transformations Solver

- The residual for bone $\hat{j}$

$$E^t = \left\| v_i^t - \sum_{j=1}^{|B|} w_{ij}(R_j^t p_i + T_j^t) \right\|^2$$

Bone Transformations Solver

- The residual q_i^t for bone $\hat{j}$

$$E^t = \left\| v_i^t - \sum_{j=1}^{|B|} w_{ij}(R_j^t p_i + T_j^t) \right\|^2$$

$$E_{\hat{j}}^t = \sum_{i=1}^{|V|} \left\| \underbrace{v_i^t - \sum_{j=1, j\neq\hat{j}}^{|B|} w_{ij}(R_j^t p_i + T_j^t)}_{q_i^t} - \underbrace{w_{i\hat{j}}(R_{\hat{j}}^t p_i + T_{\hat{j}}^t)}_{\text{Bone } \hat{j} \text{ out}} \right\|^2$$

Bone Transformations Solver

- The residual q_i^t for bone $\hat{j}$

$$E^t = \left\| v_i^t - \sum_{j=1}^{|B|} w_{ij}(R_j^t p_i + T_j^t) \right\|^2$$

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Bone $\hat{j}$ out

- Now find rigid transformation

$$p_i \xrightarrow{(R_{\hat{j}}^t,\, T_{\hat{j}}^t)} q_i^t$$

Bone Transformations Solver

- Remove the translation

$$\overline{p}_i = p_i - p_*$$
$$\overline{q}_i^t = q_i^t - w_{i\hat{j}} q_*^t$$

Center of Rotation:

$$p_* = \frac{\sum_{i=1}^{|V|} w_{i\hat{j}}^2 p_i}{\sum_{i=1}^{|V|} w_{i\hat{j}}^2}$$

$$q_*^t = \frac{\sum_{i=1}^{|V|} w_{i\hat{j}} q_i^t}{\sum_{i=1}^{|V|} w_{i\hat{j}}^2}$$

Bone Transformations Solver

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- Calculate the rotation by Singular Value Decomposition

Compare Equations

- Remove the translation

Our method	Weighted Absolute Orientation
$\overline{p}_i = p_i - p_*$ $\overline{q}_i^t = q_i^t - w_{i\hat{j}} q_*^t$	$\overline{p}_i = p_i - p_*$ $\overline{q}_i^t = q_i^t - q_*^t$
Center of Rotation:	Center of Rotation:
$p_* = \frac{\sum_{i=1}^{\vert V\vert} w_{i\hat{j}}^2 p_i}{\sum_{i=1}^{\vert V\vert} w_{i\hat{j}}^2}$	$p_* = \frac{\sum_{i=1}^{\vert V\vert} w_{i\hat{j}} p_i}{\sum_{i=1}^{\vert V\vert} w_{i\hat{j}}}$
$q_*^t = \frac{\sum_{i=1}^{\vert V\vert} w_{i\hat{j}} q_i^t}{\sum_{i=1}^{\vert V\vert} w_{i\hat{j}}^2}$	$v_*^t = \frac{\sum_{i=1}^{\vert V\vert} w_{i\hat{j}} v_i^t}{\sum_{i=1}^{\vert V\vert} w_{i\hat{j}}}$

- Calculate the rotation by Singular Value Decomposition

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- Remove the translation

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$\overline{q}_i^t = q_i^t - w_{i\hat{j}} q_*^t$	$\overline{q}_i^t = q_i^t - q_*^t$
Center of Rotation:	Center of Rotation:
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Why there are differences?

Compare Equations

Our method	Weighted Absolute Orientation
$E^t = \left\| v_i^t - \sum_{j=1}^{\|B\|} w_{ij} (R_j^t p_i + T_j^t) \right\|^2$	$E^t = \sum_{j=1}^{\|B\|} \sum_{i=1}^{\|V\|} w_{ij} \left\| v_i^t - (R_j^t p_i + T_j^t) \right\|^2$

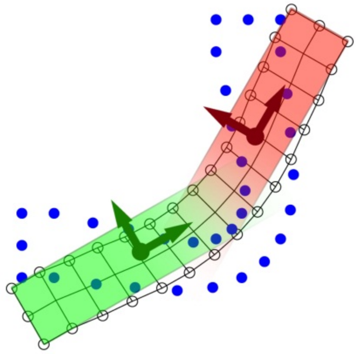
Different objective functions!

Compare Equations

Our method	Weighted Absolute Orientation
$E^t = \left\| v_i^t - \sum_{j=1}^{\|B\|} w_{ij} (R_j^t p_i + T_j^t) \right\|^2$	$E^t = \sum_{j=1}^{\|B\|} \sum_{i=1}^{\|V\|} w_{ij} \left\| v_i^t - (R_j^t p_i + T_j^t) \right\|^2$
Bone blending	No blending

Different objective functions!

Toy Example

Our method	Weighted Absolute Orientation
	

Toy Example

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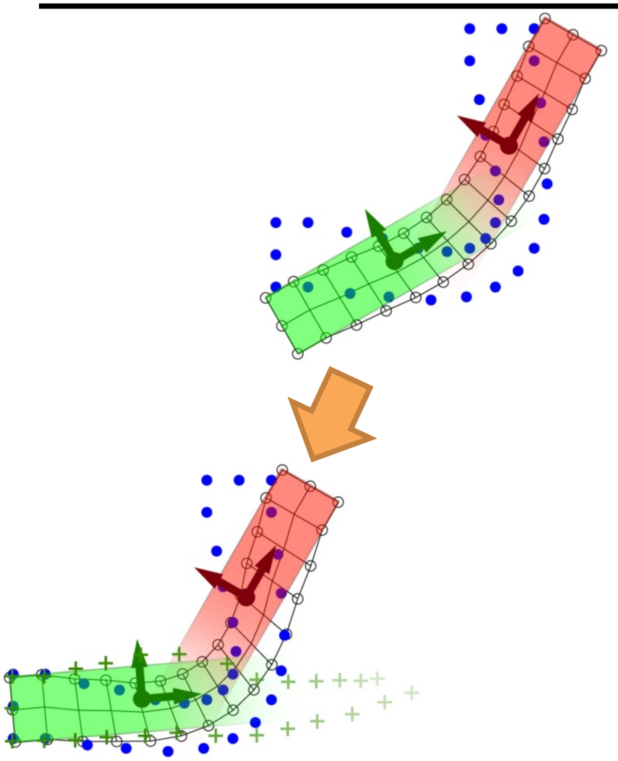

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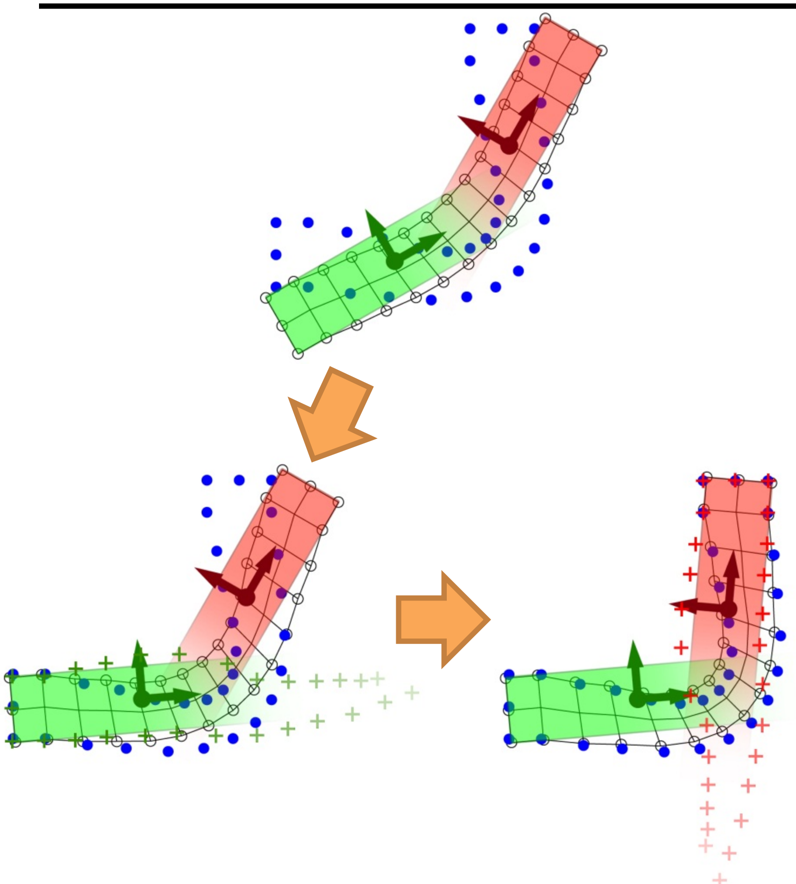
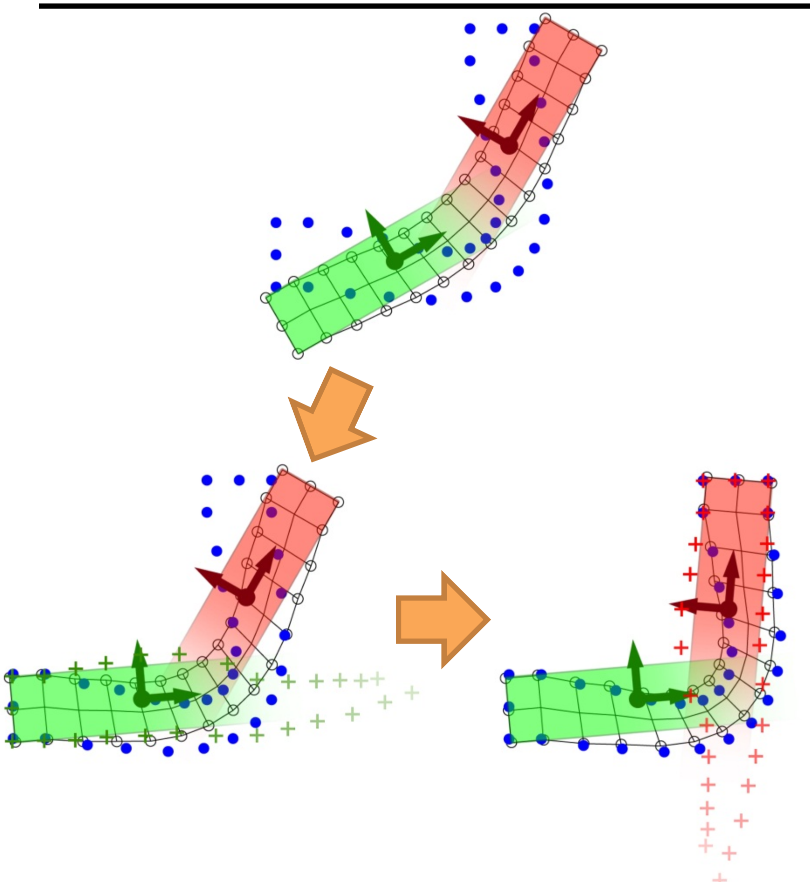
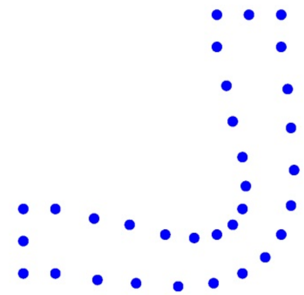


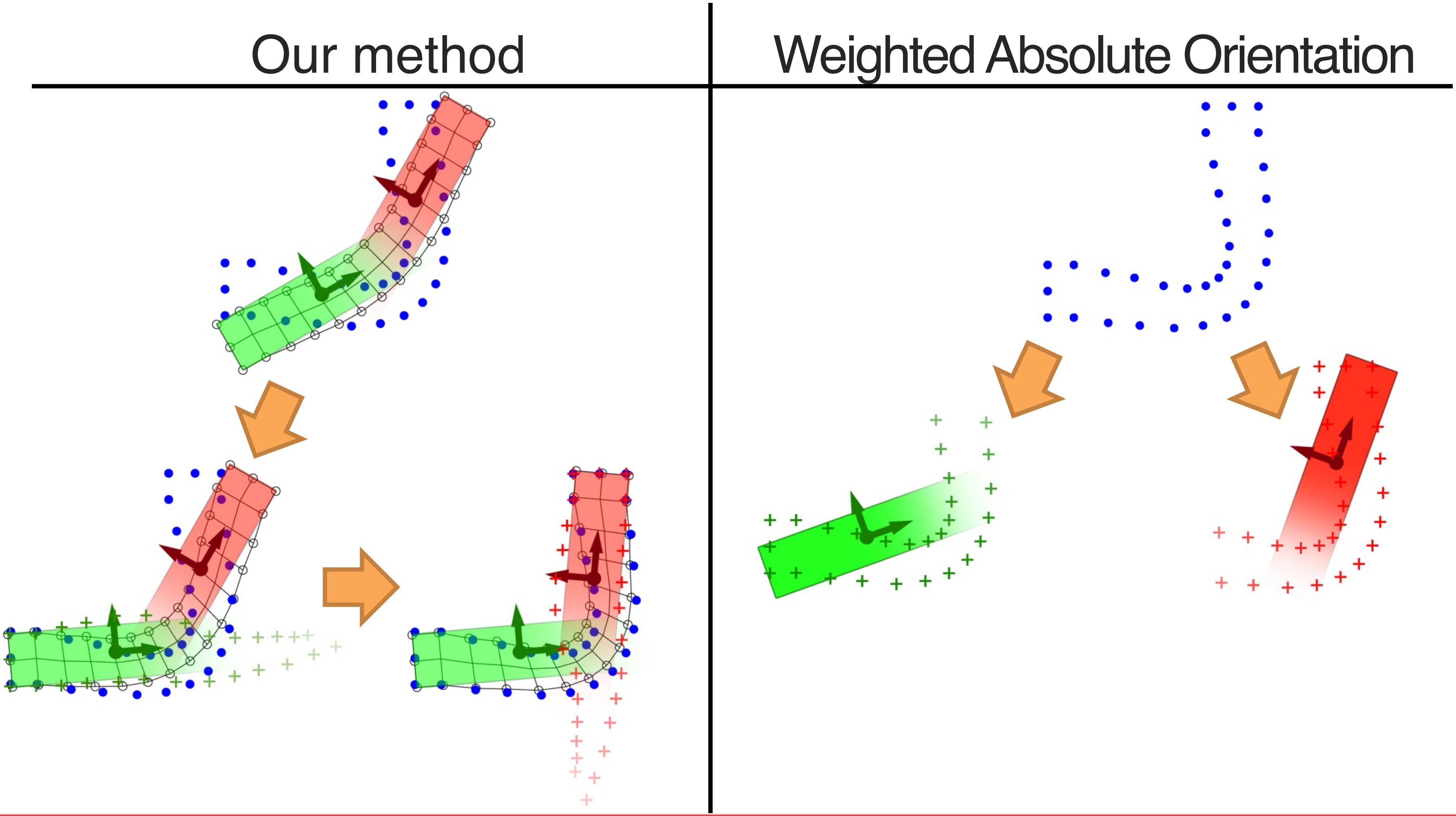
Toy Example

Our method | **Weighted Absolute Orientation**





Toy Example



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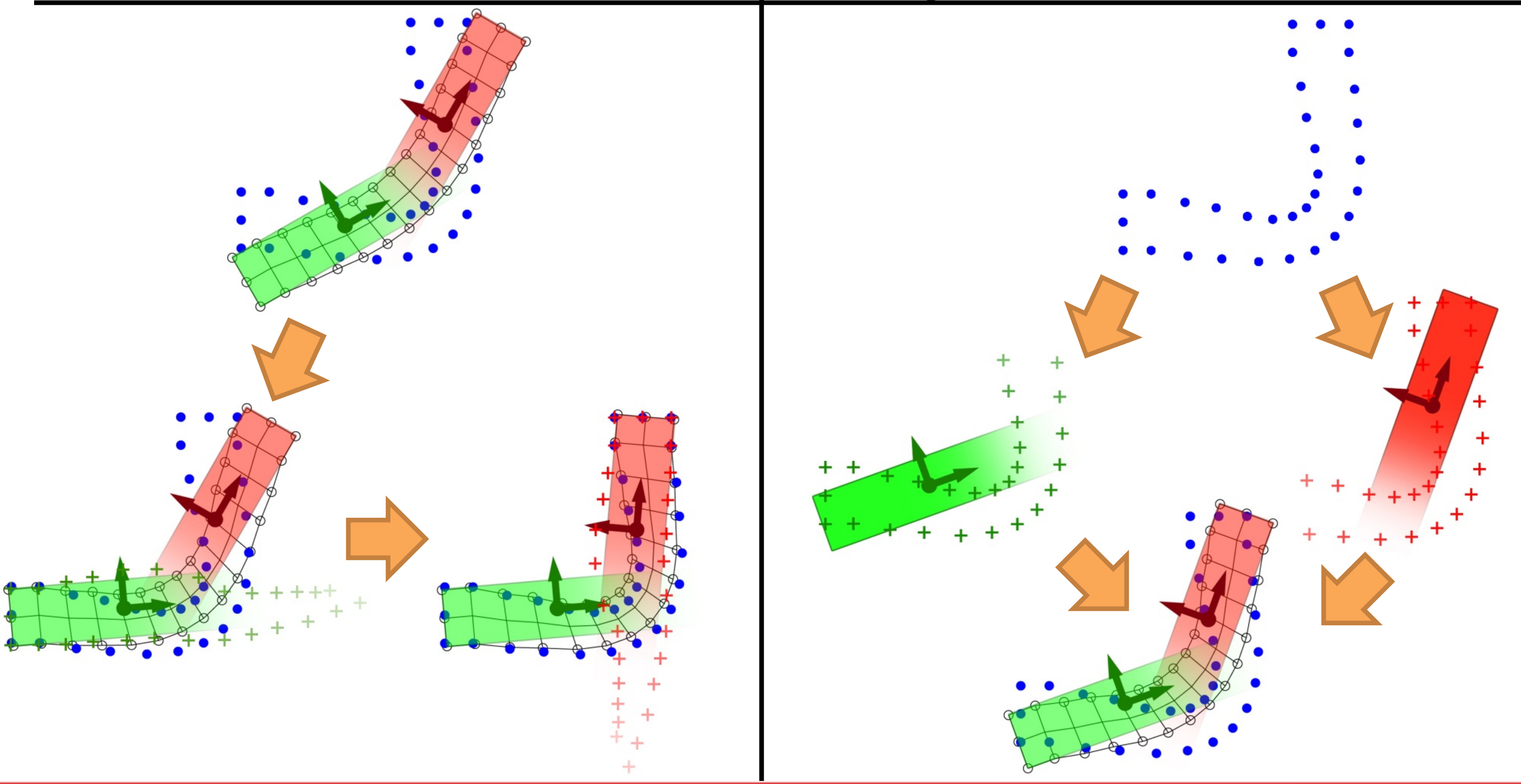

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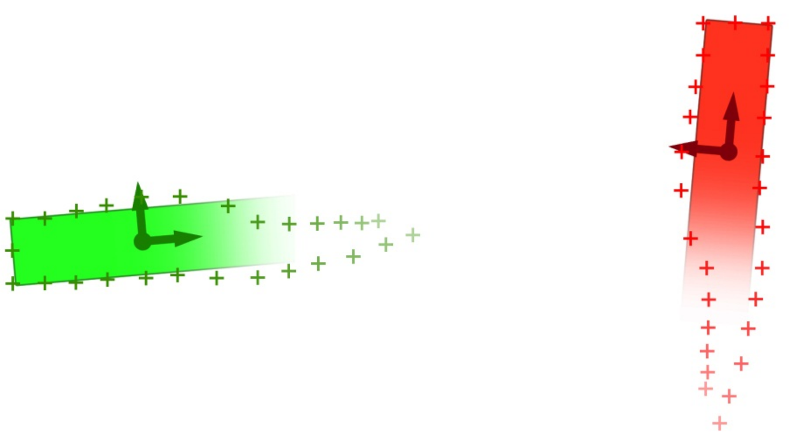

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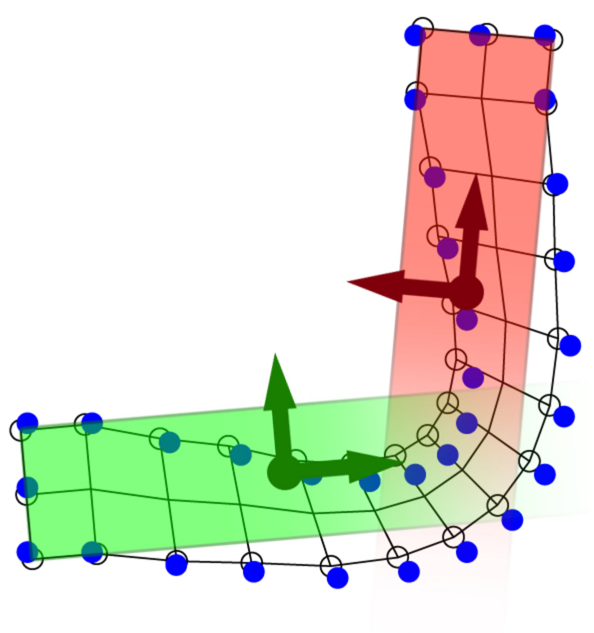

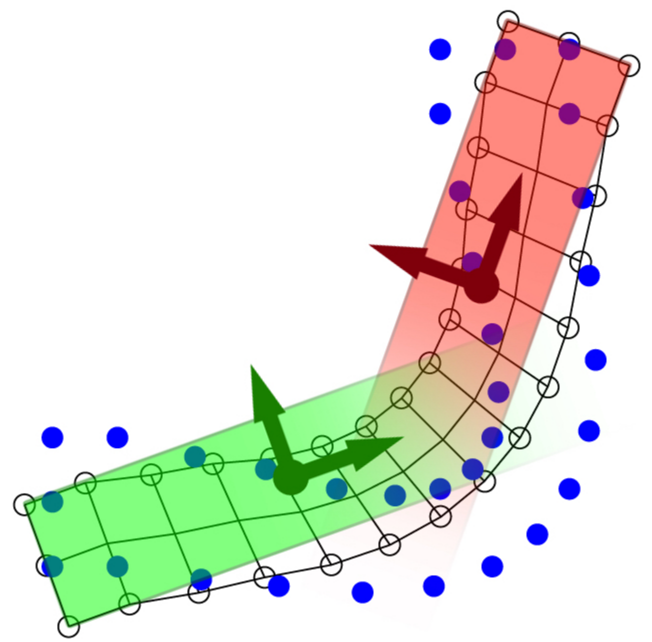

Bone Transformations Re-Initialization

- Recall: Center of Rotation

$$p_* = \frac{\sum_{i=1}^{|V|} w_{i\hat{j}}^2 p_i}{\sum_{i=1}^{|V|} w_{i\hat{j}}^2}$$

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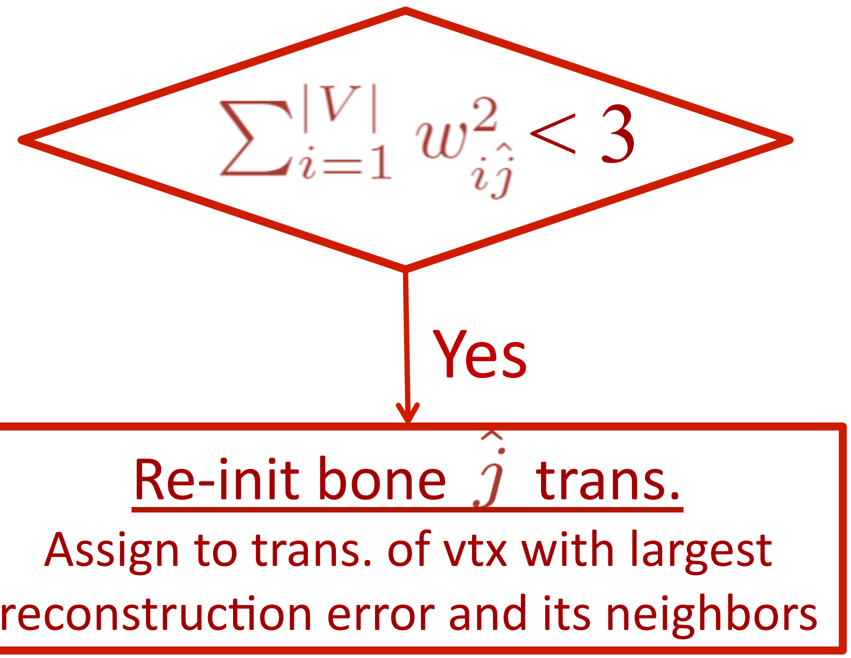

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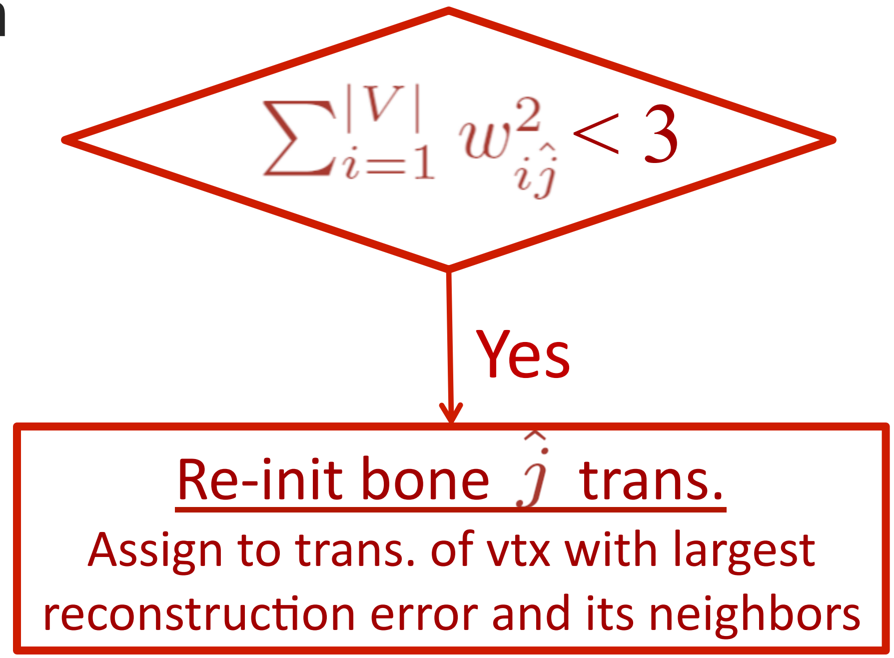
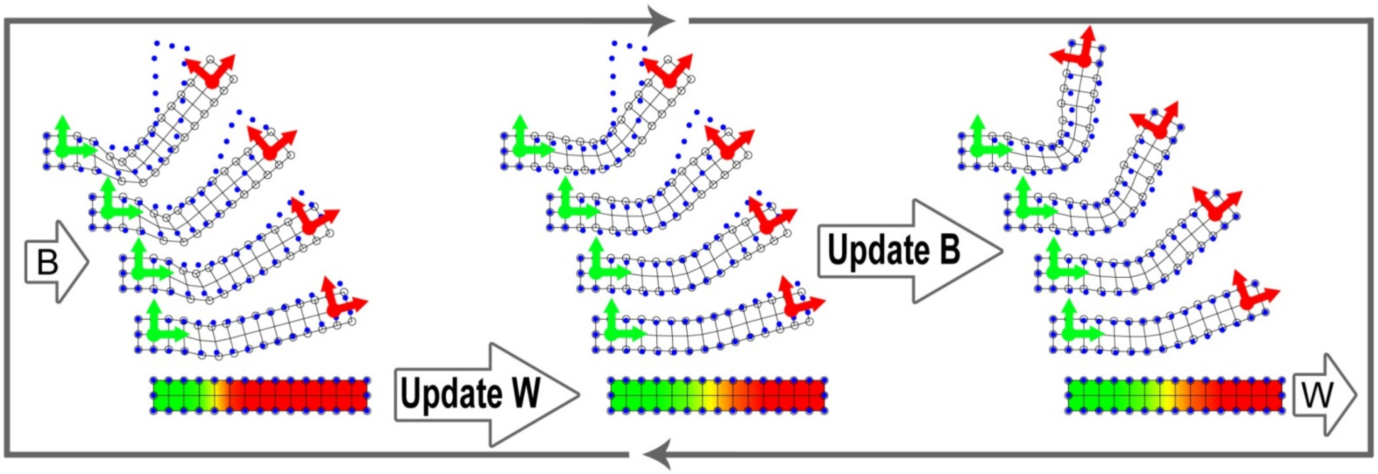


- Improve stability
- Jump out of local minimums

Summary of Our Algorithm



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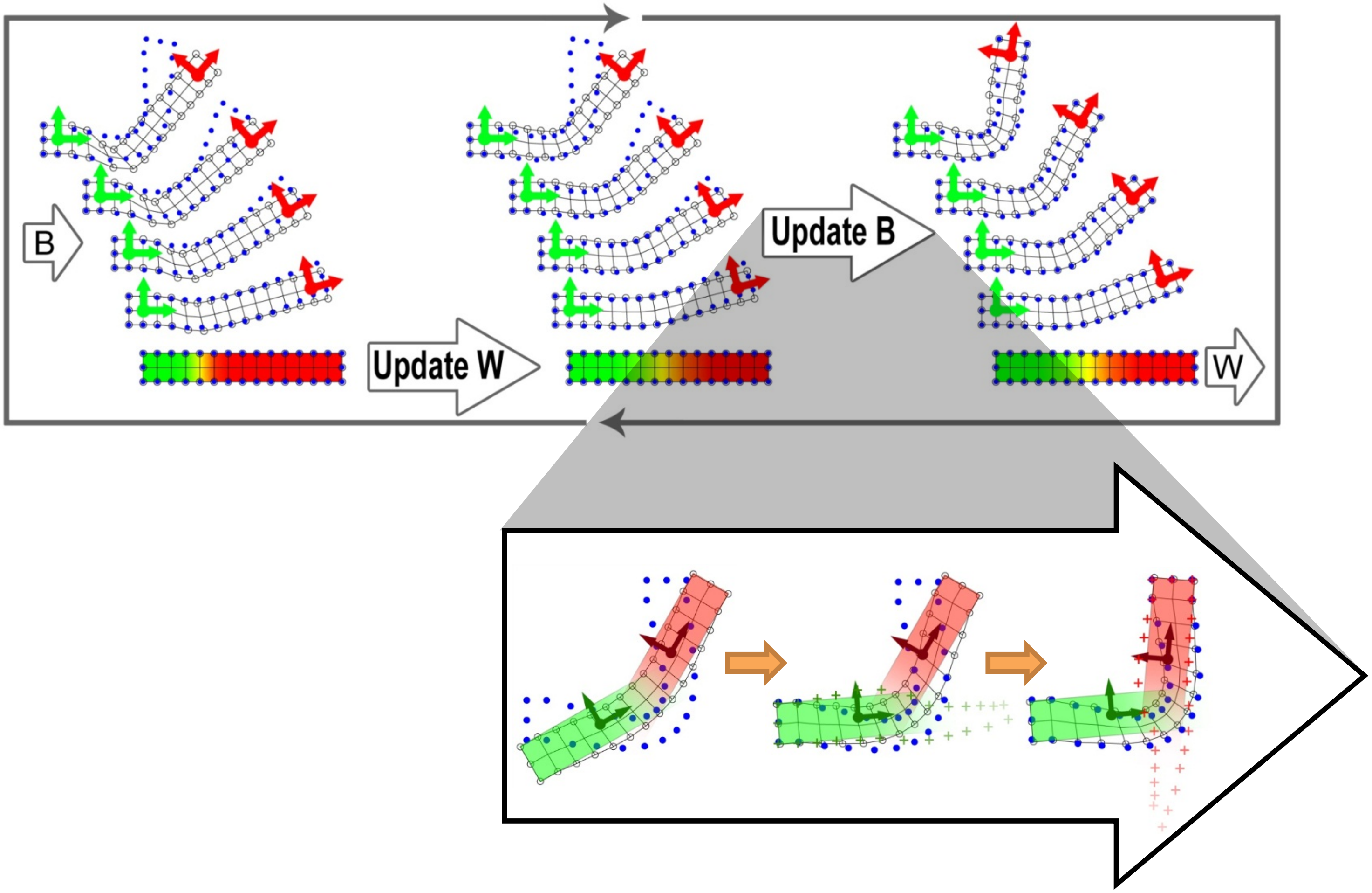
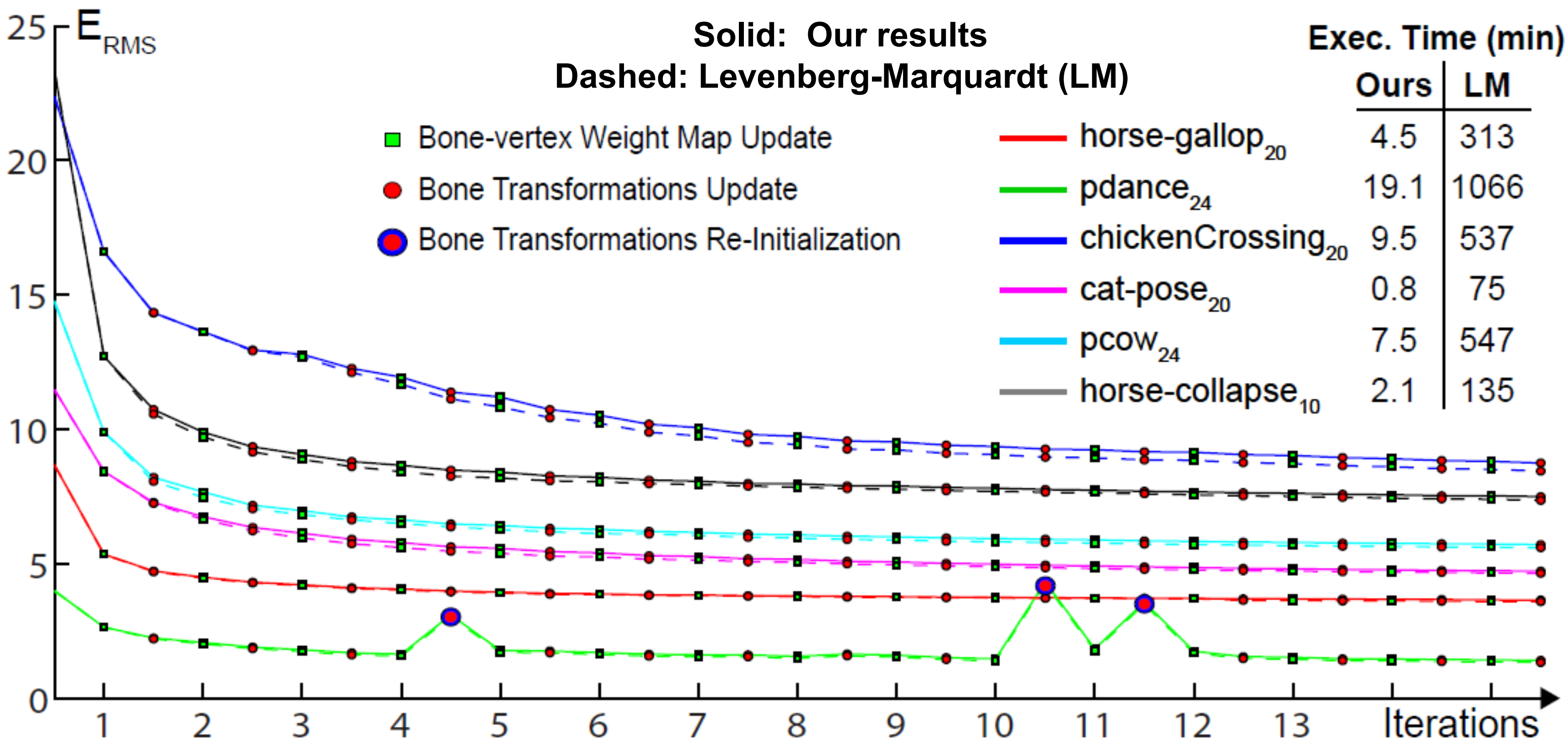


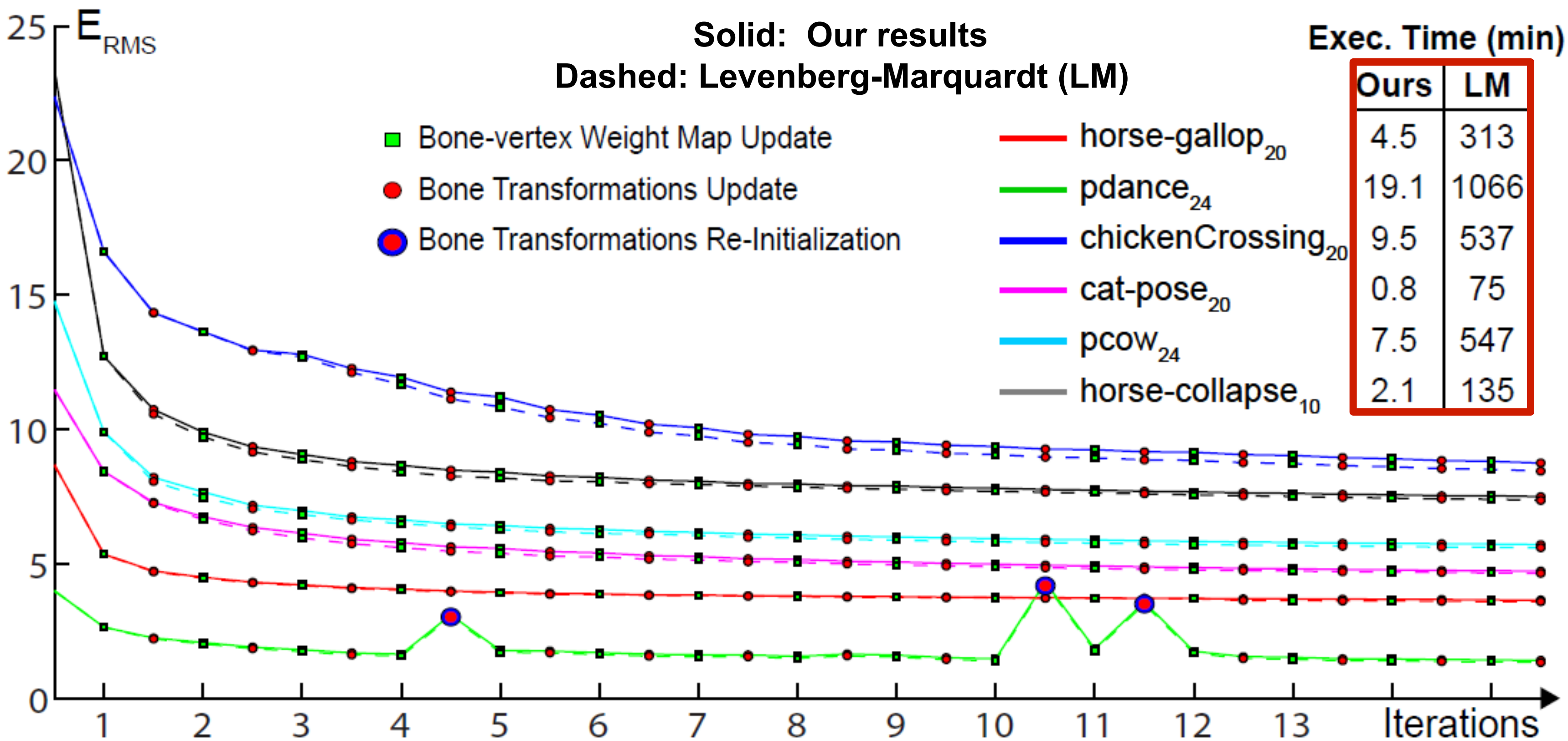
Convergence



	Exec. Time (min) Ours	LM
horse-gallop$_{20}$	4.5	313
pdance$_{24}$	19.1	1066
chickenCrossing$_{20}$	9.5	537
cat-pose$_{20}$	0.8	75
pcow$_{24}$	7.5	547
horse-collapse$_{10}$	2.1	135

- Error decreases monotonically (without re-init bone transformations)

Convergence



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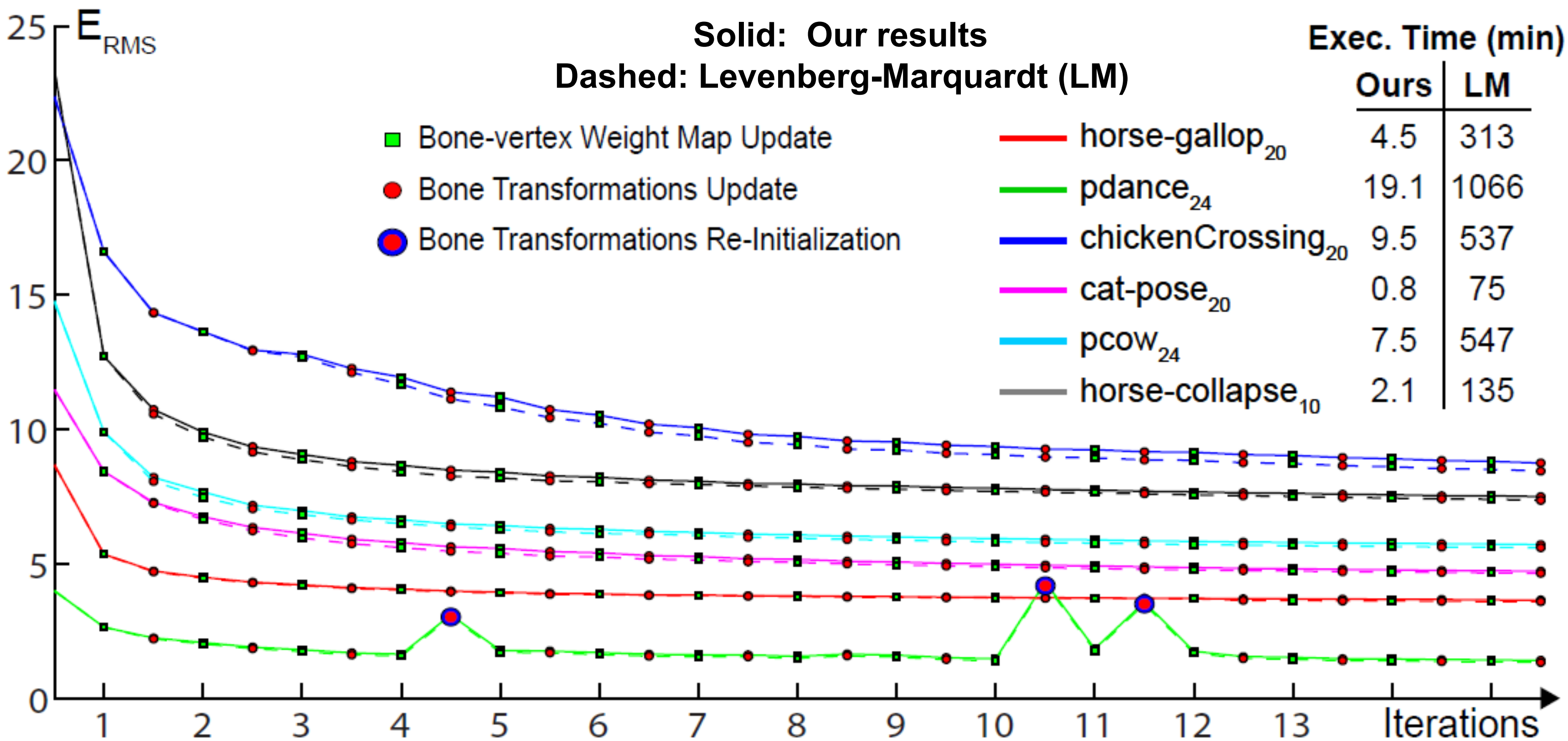
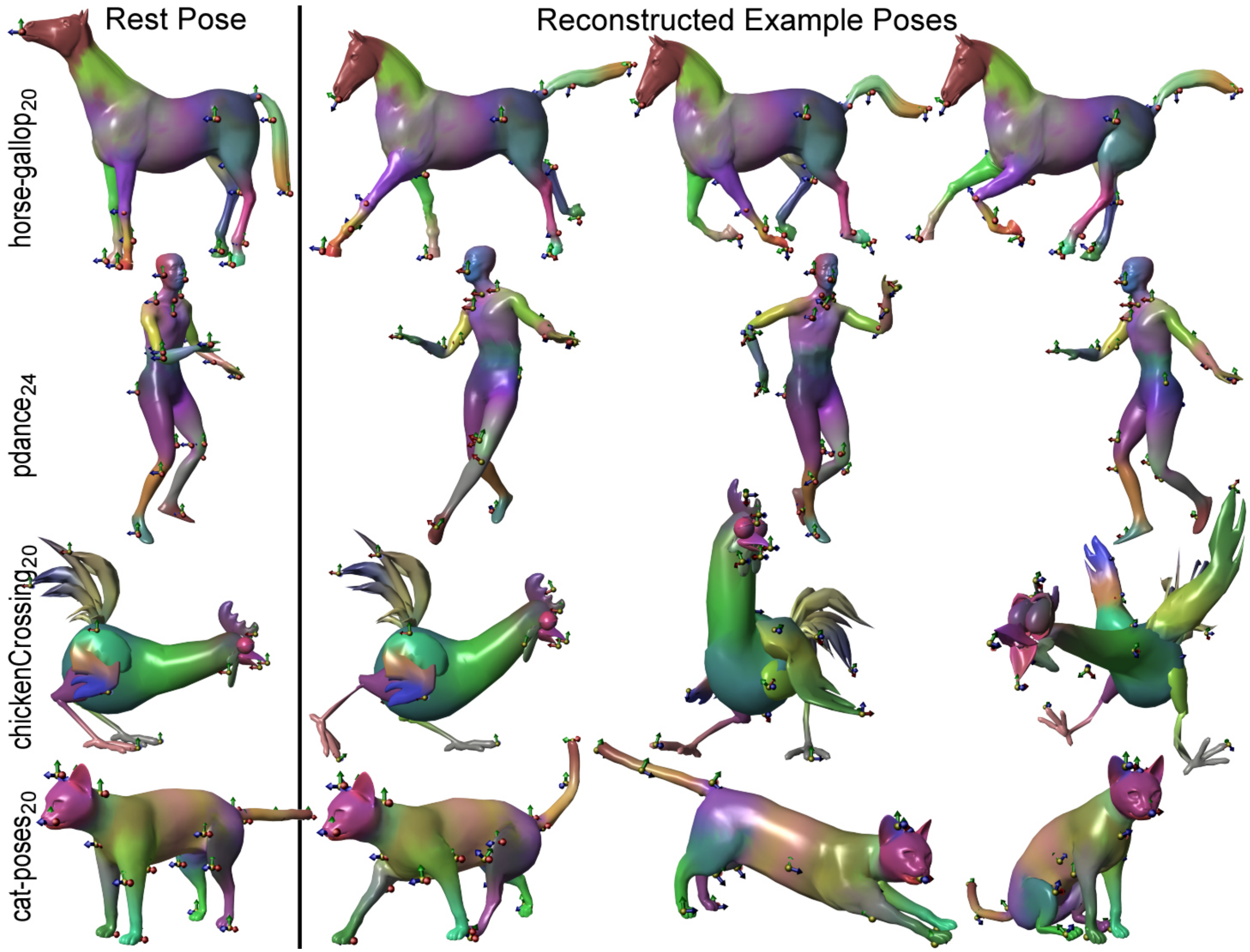
- Our algorithm converges much faster than LM (~50 times)

Convergence

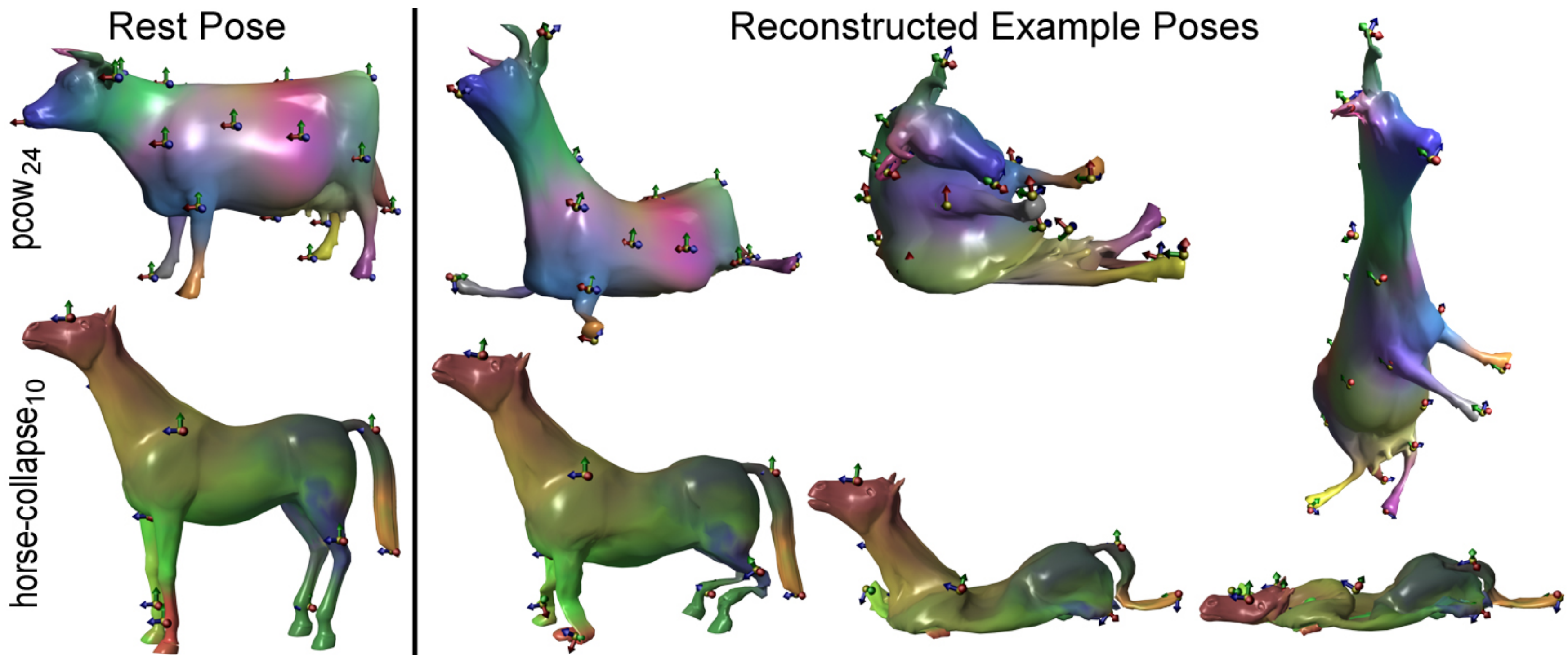


- Error decreases monotonically (without re-init bone transformations)
- Our algorithm converges much faster than LM (~50 times)
- One pass is enough for bone transformations update

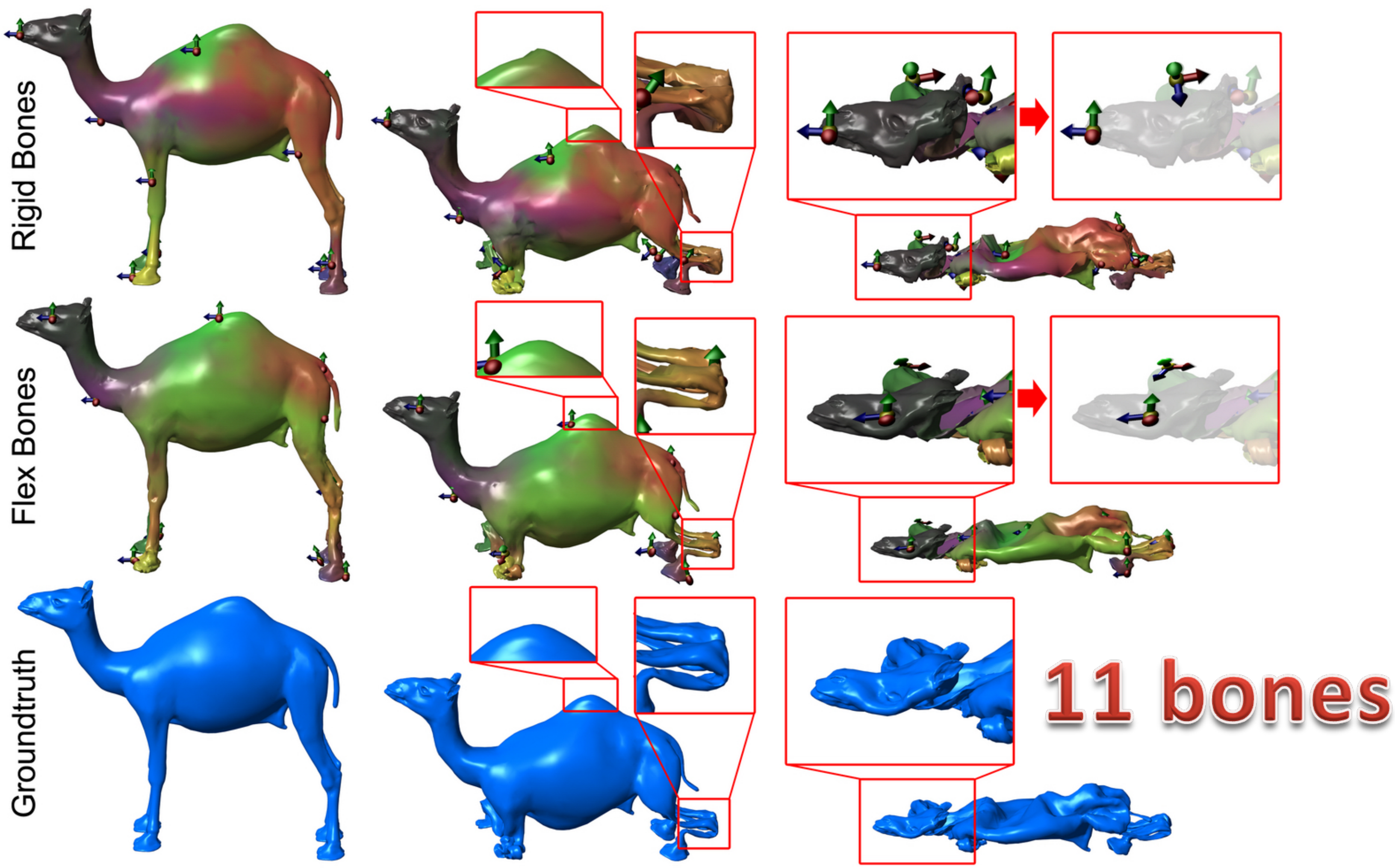
Results – Articulated models



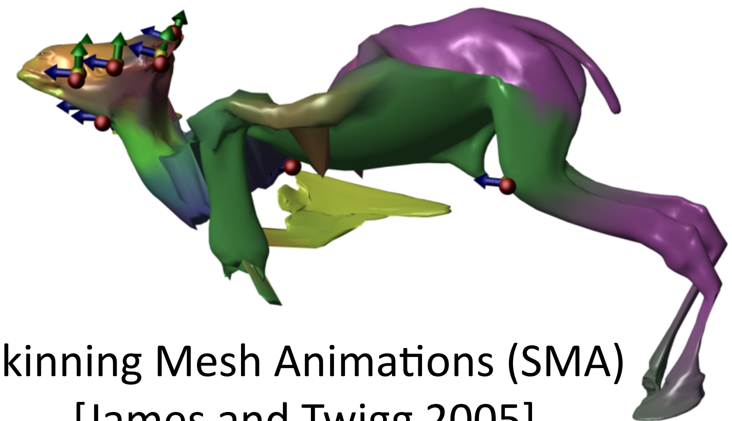
Results – Highly deformable models



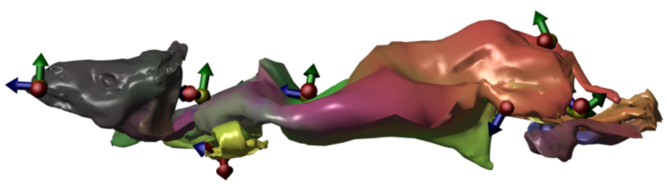
Rigid Bones vs. Flexible Bones



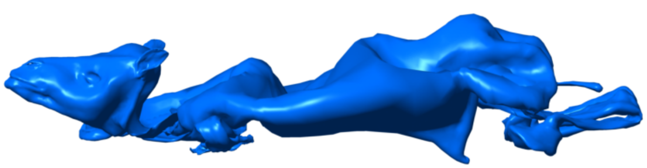
Comparison



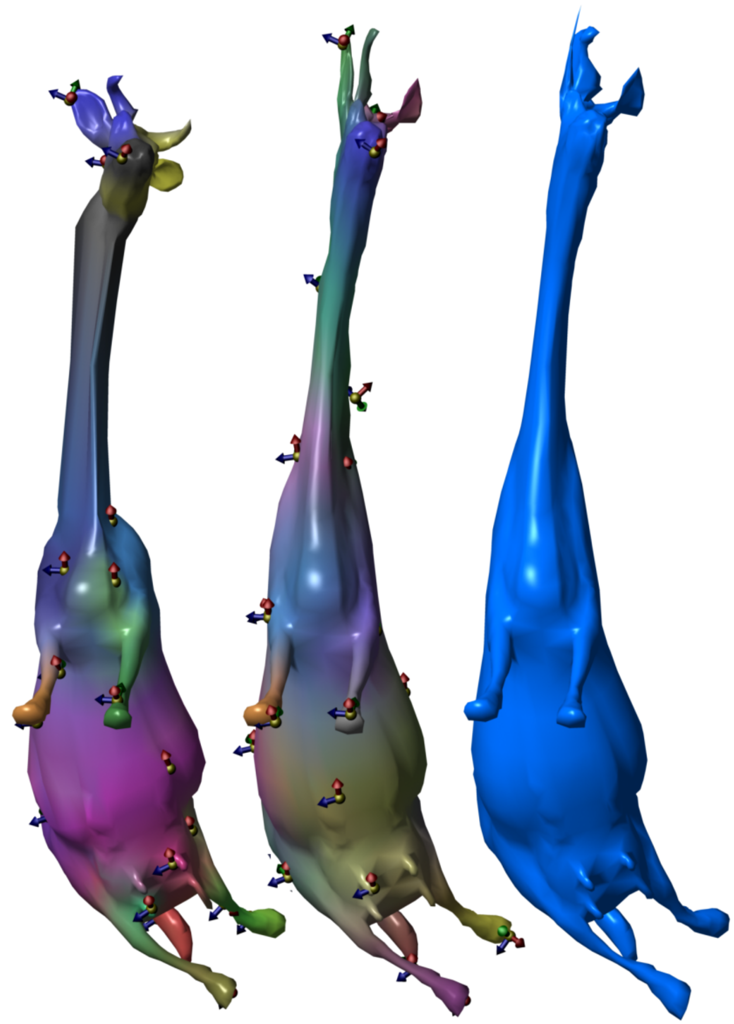
Skinning Mesh Animations (SMA)
[James and Twigg 2005]



Our method

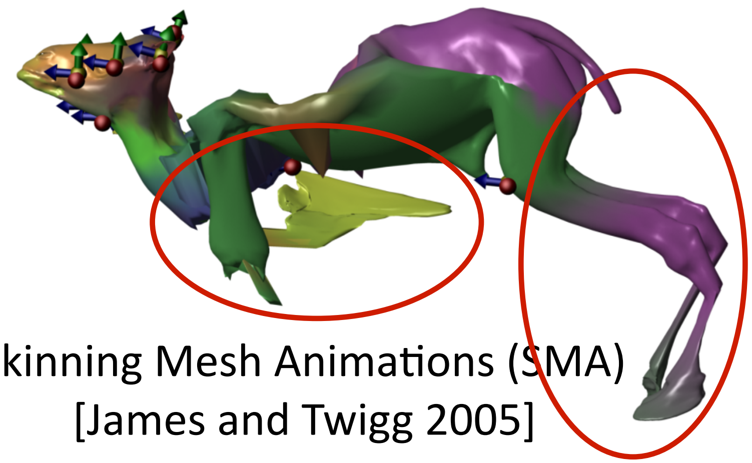


Ground truth

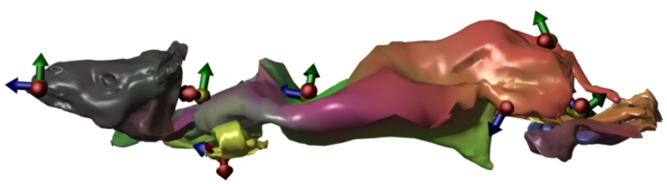


SMA Ours Ground truth

Comparison



Skinning Mesh Animations (SMA)
[James and Twigg 2005]



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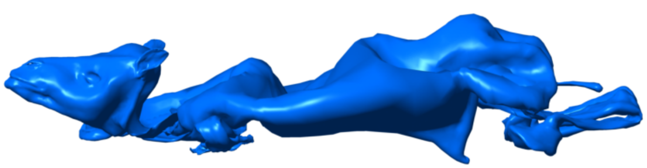

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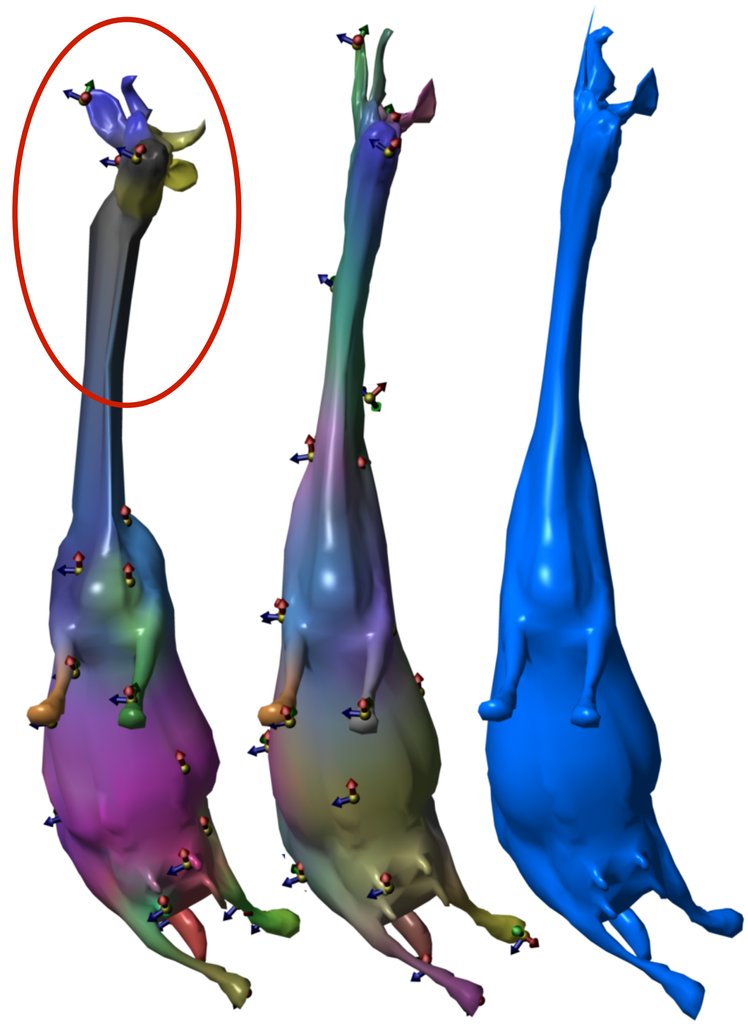

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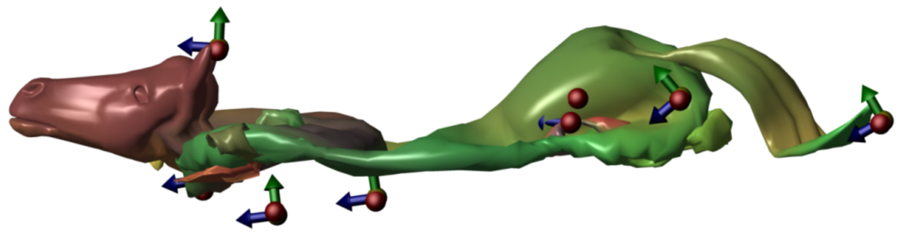
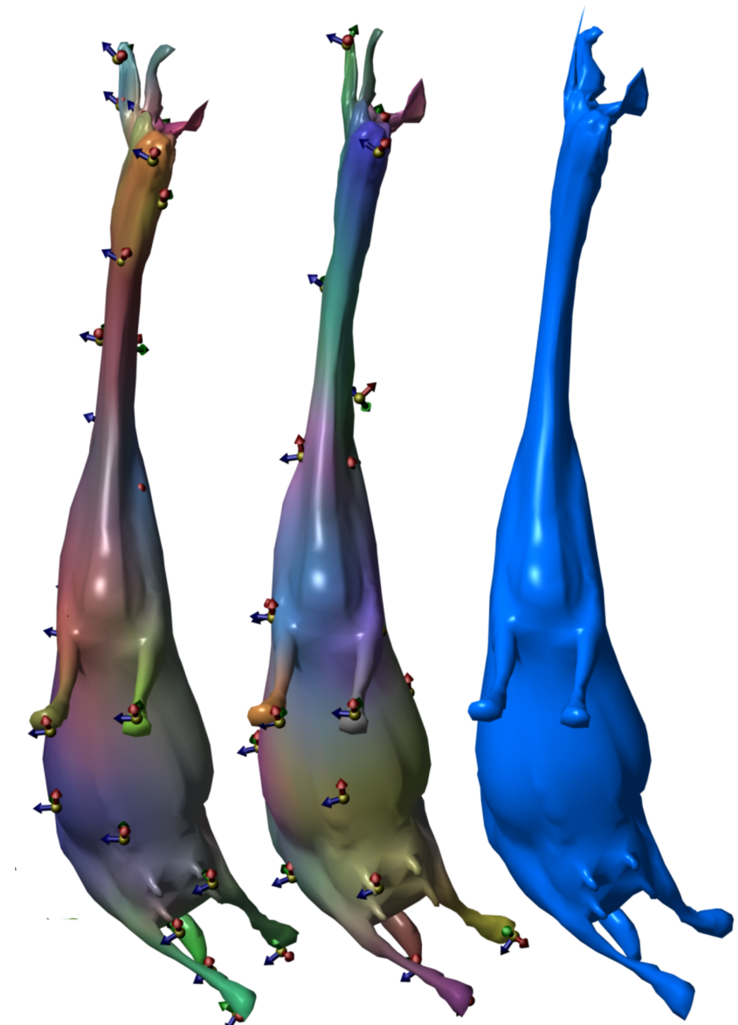


Learning Skeletons for Shape and Pose (LSSP) [Hasler et al. 2010]

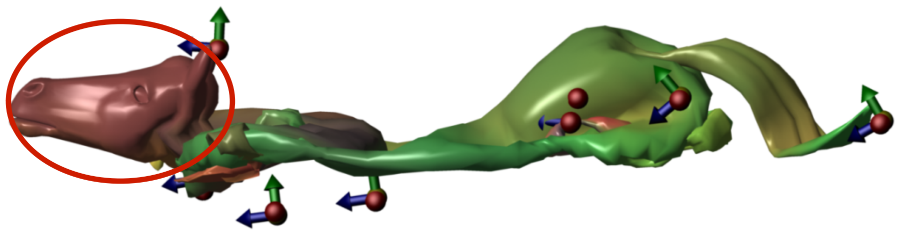
Our method

Ground truth



LSSP Ours Ground truth

Comparison



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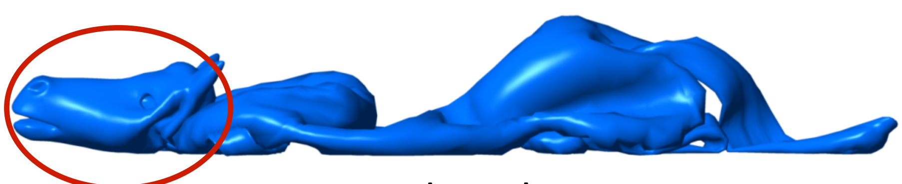
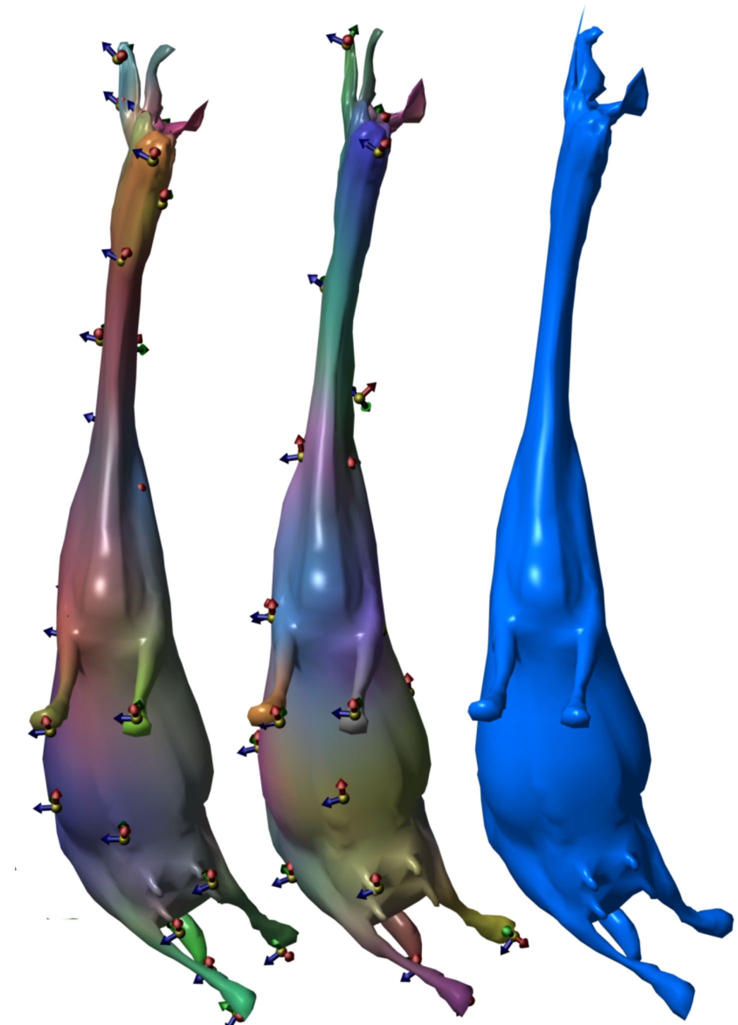


Ground truth



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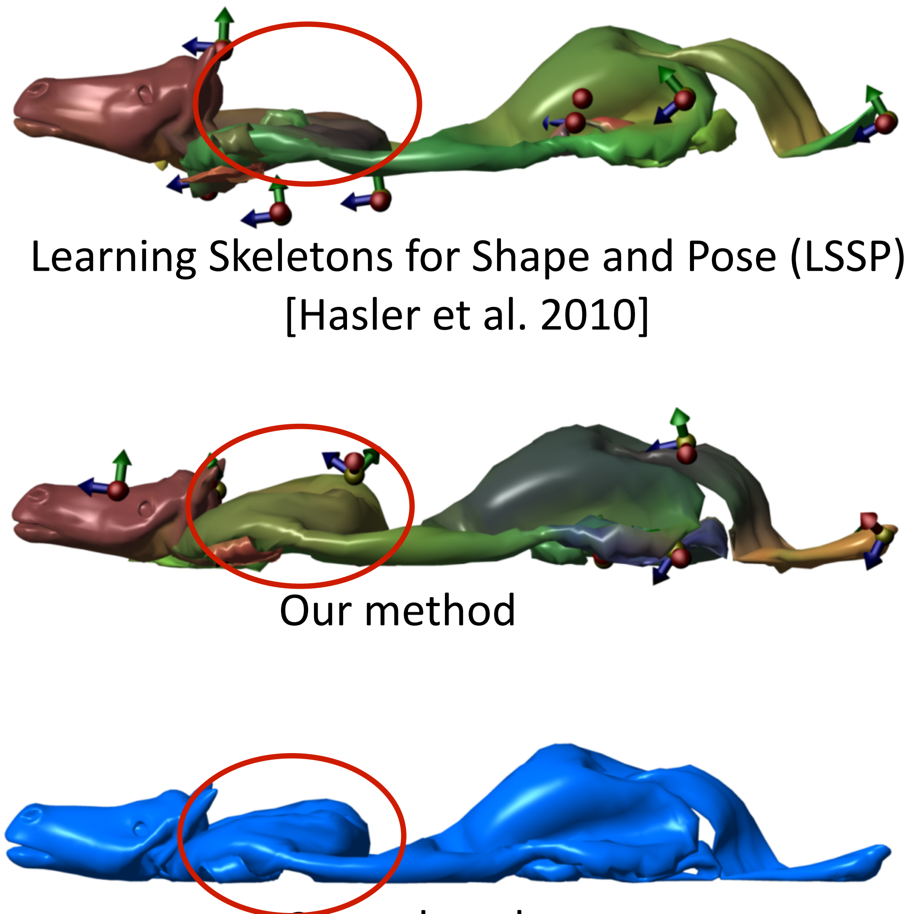

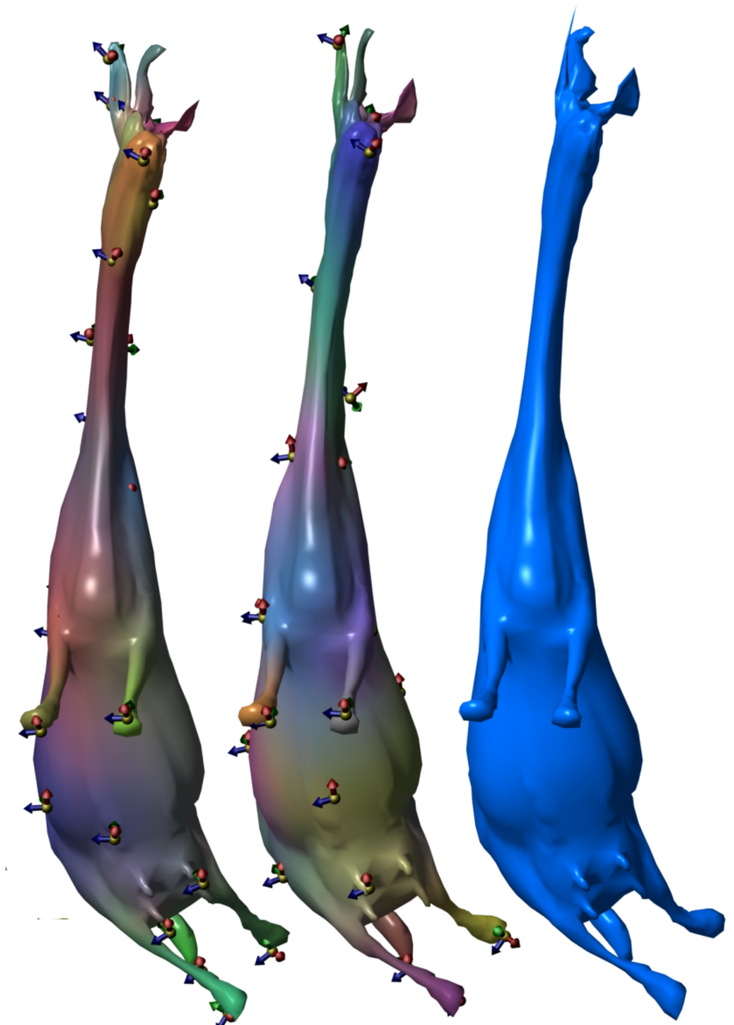

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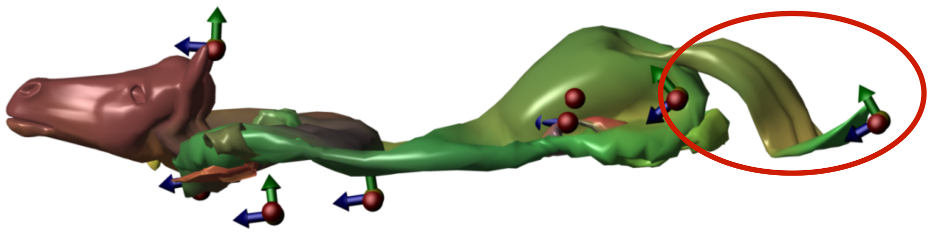

Learning Skeletons for Shape and Pose (LSSP)
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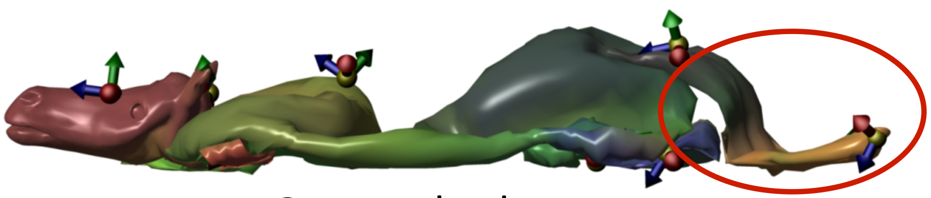

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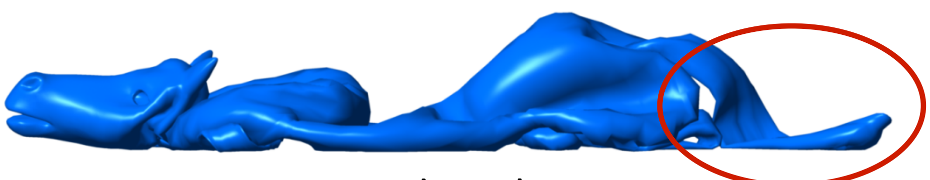

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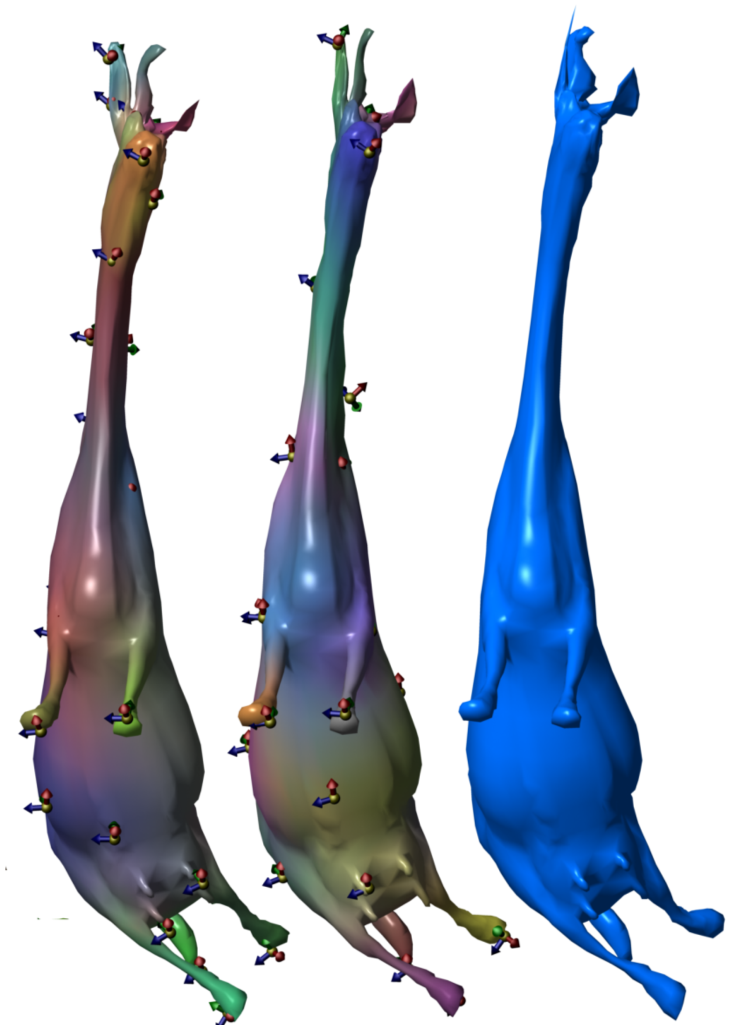

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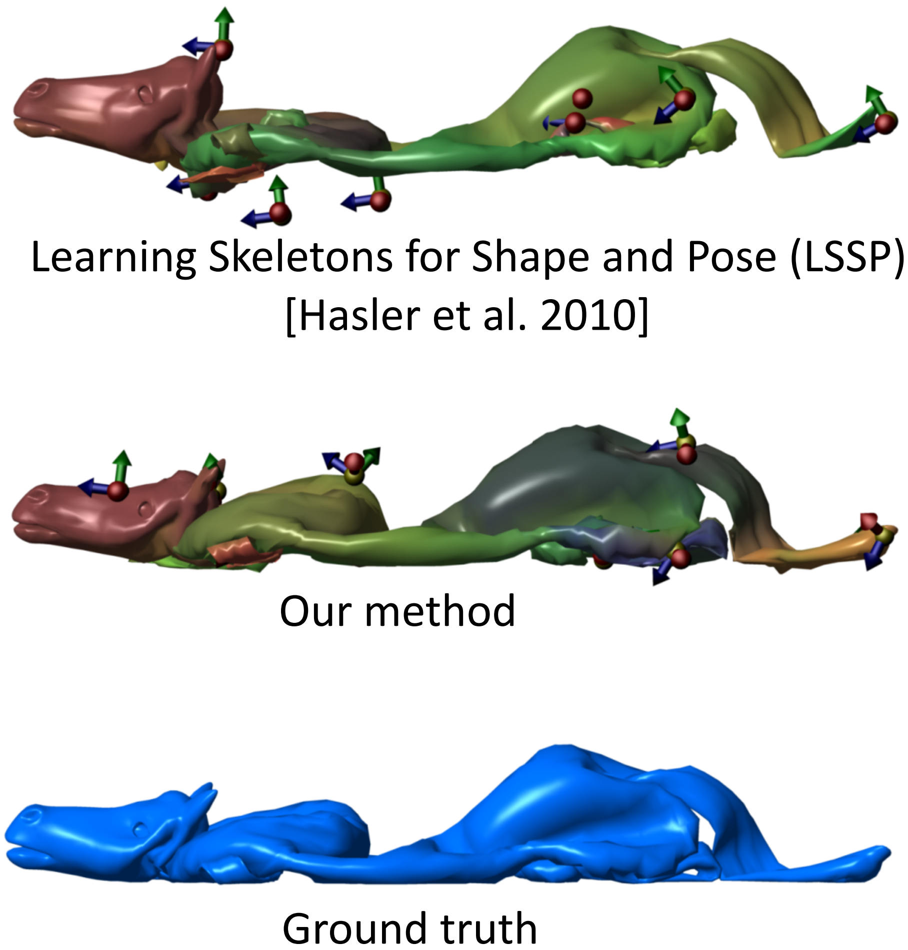

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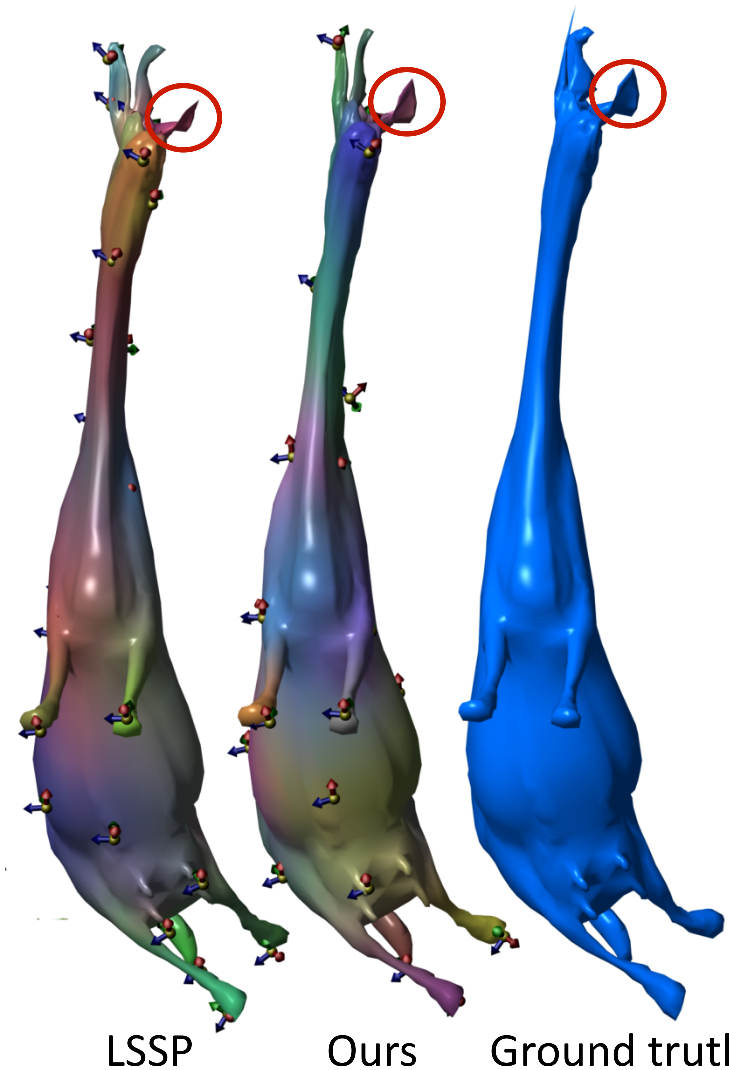

LSSP
Ours
Ground truth

Comparison

Dataset$_{[\text{No. of bones}]}$	Approximation error E_{RMS}			Execution time (minutes)		
	SMA	LSSP	SSDR	SMA	LSSP	SSDR
camel-collapse$_{11}$	125.3 (4)	-	5.4(1.7)	13.8	-	7.4
cat-poses$_{25}$	8.5 (3.1)	6.2(3.3)	3.4(1.4)	0.7	371.7	1.5
chickenCrossing$_{28}$	12.5 (4.2)	6.2(5.1)	8.1(5.4)	14.1	1165.4	24
horse-gallop$_{33}$	9.5 (1.5)	12.5(4.6)	2.2(1.1)	3.8	911	9.8
lion-poses$_{21}$	62.8 (5.7)	7.7(3.9)	4.4(2.2)	0.6	360.2	0.8
pcow$_{24}$	24.8 (13.2)	7.2(6.7)	5.7(4.8)	3.8	564.5	8.9
pdance$_{24}$	3.8 (1.6)	3.4(2.3)	1.3(0.8)	22	2446.8	28.3

SMA: Skinning Mesh Animations [James and Twigg 2005]
LSSP: Learning Skeletons for Shape and Pose [Hasler et al. 2010]
SSDR: Smooth Skinning Decomposition with Rigid Bones (our method)
Result in parentheses: rank-5 EigenSkin correction [Kry et al. 2002]

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We are about 100 times faster!

Conclusion

Linear Blend Skinning Decomposition Model

- Convex, sparse weights

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- Convex, sparse weights

✓ Rigid bone transformations

✓ Iterative bone transformation linear solvers

  ✓ Nearly optimized, working well with highly deformation models

  ✓ Fast

  ✓ Simple

Conclusion

Linear Blend Skinning Decomposition Model

- Convex, sparse weights

✓ Rigid bone transformations

✓ Iterative bone transformation linear solvers

  ✓ Nearly optimized, working well with highly deformation models

  ✓ Fast

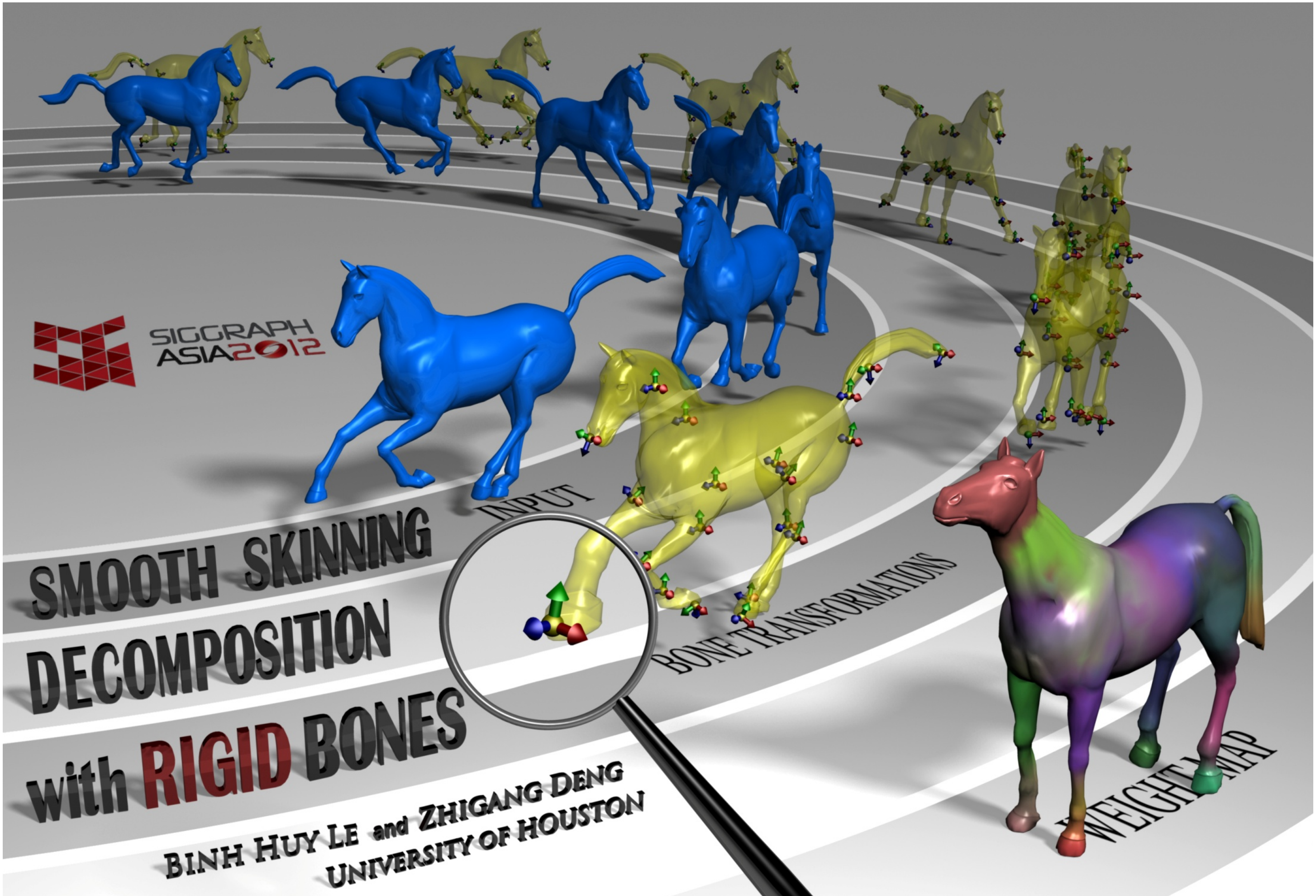
  ✓ Simple

✗ Considering skeleton hierarchy

✗ Utilizing other information: Mesh topology or anatomy

Acknowledgements

- NSF IIS-0915965
- Vietnam Education Foundation (VEF)
- Google and Nokia for research gifts
- Robert Sumner, Jovan Popovic, Hugues Hoppe, Doug James, and Igor Guskov for publishing the mesh sequences
- Anonymous reviewers for giving insightful comments



http://graphics.cs.uh.edu/ble/papers/2012sa-ssdr/

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