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## Shadow traffic: A unified model for abnormal traffic behavior simulation

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### ABSTRACT

Abnormal traffic behaviors are common traffic phenomena. Existing traffic simulators focus on showing how traffic flow develops after an anomaly occurs; however, they cannot depict the anomaly itself. In this paper, we introduce the concept of shadow traffic for modeling traffic anomalies in a unified way in traffic simulations. We transform the properties of anomalies to the properties of shadow vehicles and then describe how these shadow vehicles participate in traffic simulations. Our model can be incorporated into most existing traffic simulators with little computational overhead. Moreover, experimental results demonstrate that our model is capable of simulating a variety of abnormal traffic behaviors realistically and efficiently.

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### 1. Introduction

Abnormal traffic behaviors describe behaviors of points on road networks which can be identified as irregular behaviors from normal ones [1–5]. They deviate from ordinary types and are not developed by their downstream traffic flows, such as road breakdowns, crash accidents, pedestrian and vehicle interactions, etc. Abnormal traffic behaviors are common phenomena in traffic systems, especially in bad weather, congested areas, and hybrid traffic, etc. Modeling these behaviors easily and naturally has direct influence on the effectiveness of virtual traffic simulations.

Existing traffic simulations focus on showing different kinds of traffic flows that can exist in different scenarios [6–16]. These simulations can give detailed reconstructions and representations about how the flow develops upstream of the place where a traffic anomaly happens, but they cannot depict the anomaly itself. Some methods in the pattern recognition and transportation fields were presented for traffic anomaly simulations [2–4,15,17–21]. However, they are primarily designed for traffic analysis: predicting and exploring the cause of anomalies. Therefore, most existing traffic simulation systems do not have anomaly generation and editing functions [22, 23, 24]. Users must generate and edit the anomaly indirectly by modifying some road and traffic elements, such as a lane closure, which is complicated. Moreover, these systems cannot

give a detailed anomaly representation on their three-dimensional presentations.

In this paper, we introduce a novel concept, shadow traffic, to model a variety of abnormal traffic behaviors in a unified way. The shadow traffic is the hypothetical traffic at the abnormal point, where vehicles may not exist. We adopt the behaviors of shadow vehicles to model abnormal traffic behaviors (The red and translucent vehicles shown in Fig. 1 are shadow vehicles). Anomalies then participate in a traffic simulation through shadow traffic. In other words, we inject intangible factors into traffic simulations by embodying them in “physical” form and relying on the simulator’s pre-existing functionality.

In our shadow traffic model, we firstly present a semantic description of an anomaly in a unified way, in which we use three intensities, including spatial intensity, time intensity, and state intensity, to depict the anomaly. We then introduce shadow vehicles into our model and transform these three intensities to the states of shadow vehicles. We present efficient algorithms to compute the states of shadow vehicles. We at last realize a traffic simulation containing both shadow vehicles and non-shadow vehicles. The shadow traffic formulation provides an elegant method for an anomaly to extend its influence to the traffic flow. A variety of anomalies can be easily depicted as the states of shadow vehicles.

The main contributions of this work are as follows: Firstly, we creatively map abnormal behaviors to the states of shadow vehicles and introduce a concept model to model a variety of abnormal traffic behaviors in a unified way; Secondly, we present a traffic simulation framework including an existing traffic flow model and

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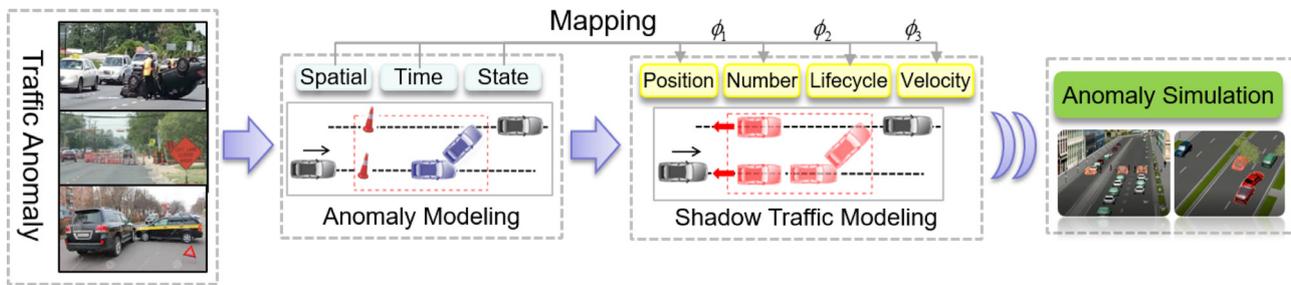


Fig. 1. The framework of our shadow traffic model.

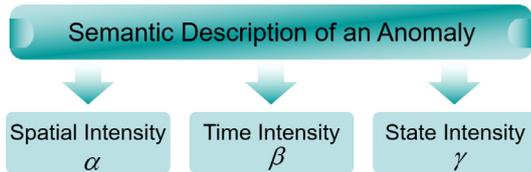


Fig. 2. The contents of the anomaly's semantic description.

our shadow traffic model, which can give realistic simulations both about traffic anomalies themselves and how traffic flow around them evolve. Last but not the least, our shadow traffic model has a high runtime efficiency. The computing time increases slightly when anomaly regions become larger. The runtime overhead of adding our shadow traffic model to traffic simulators is negligible.

To demonstrate the merits of our model, we give the efficiency analysis of our model in a traffic simulation system. We also show that it is capable of modeling commonly observed abnormal traffic behaviors, such as road breakdowns, crashes, unexpected breaking, vehicle-pedestrian intersections, etc.

## 2. Related work

With the increasing volumes of traffic data and software tools capable of modeling urban scenes, numerous efforts have been devoted to traffic simulations.

Since Sewall et al. first introduce the concept of “virtual traffic” [25], numerous detailed models have been proposed for realistic and efficient traffic simulations, including traffic flow descriptions [7–10,13,14,26,27], traffic flow reconstructions [11,12], and mixed traffic animations [28,29]. These models focus on physically based traffic simulations; that is, they aim to give a realistic description about the evolvement of traffic flows after an anomaly occurs. They assume that the properties of the leader vehicles in an anomaly are already known, and they only model traffic flows upstream.

A considerable body of work exists in simulating traffic anomalies in the pattern recognition and transportation fields. Zhong et al. present an unsupervised technique for detecting abnormal traffic behaviors in video using many simple features [3]. Owens et al. determine whether a point on a trajectory is normal using the distributions of flow vectors [1]. Sultani et al. use the well-known intelligent driver model for detecting and localizing abnormal traffic [4]. Hu et al. present a system to automatically learn motion patterns for anomaly detection and behavior prediction based on a proposed algorithm for robustly tracking multiple objects [2]. Sabel et al. propose methods to automatically identify and report road traffic accidents [17,19]. Markus et al. present methods to model the occurrences of accidents [15,21]. Brach et al. use practical analytical techniques and dynamic methods to solve a complex vehicle anomaly reconstruction [18]. Miaou et al. discuss the relationships between vehicle accidents and highway geometric designs [20]. These models are typically designed to model whether, when and

why an anomaly occurs, but they do not describe how the anomaly itself is evolving.

To tackle the aforementioned challenges, we model traffic anomalies using shadow traffic, which is inspired by the concept of composite agents by Yeh et al. [30]. In their model, they use composite agents to model a variety of emergent behaviors in agent-based crowd simulations. They classify the agents into basic agents and composite agents. A basic agent is the agent representation native to a simulator. A composite agent is a basic agent that is associated with a set of proxy agents. The proxy agents could be thought of as hands extended from the basic agent which get extended towards other agents, encouraging those agents to step away to avoid collisions. The model is used to model the agents that have more influence than common agents in a crowd. In this paper, we introduce the idea into the modeling of abnormal traffic behaviors, which extend their influence over the upstream vehicles.

## 3. Model

In this section, we present an algorithm to model abnormal traffic behaviors by utilizing shadow vehicles. The framework of our model, as shown in Fig. 1, consists of three parts: (1) semantic description of an anomaly, (2) semantic description of shadow traffic, (3) mapping between the anomaly and shadow traffic. Each of these parts is discussed in detail below.

### 3.1. Semantic description of an anomaly

In our model, we describe a traffic anomaly by its spatial region, duration, and state. We introduce spatial intensity, time intensity, and state intensity, all of which are measurement values used for a semantic description of the anomaly, as shown in Fig. 2.

*Spatial intensity:* Spatial intensity describes the spatial region that is influenced by the traffic anomaly. It is determined by the number of lanes and the spatial region affected by the anomaly. Let  $\alpha$  be the spatial intensity, then,

$$\alpha = \bigcup_{i=1}^n \alpha_{N_i} = \bigcup_{i=1}^n [s_{N_i\_start}, s_{N_i\_end}] \quad (1)$$

Here,  $n$  is the total number of lanes affected by the anomaly.  $N_i$  is the ID number of the  $i$ th lane with  $1 \leq i \leq n$ .  $\alpha_{N_i}$  is the spatial intensity on lane  $N_i$  and  $[s_{N_i\_start}, s_{N_i\_end}]$  describes the spatial range on lane  $N_i$  affected by the anomaly.  $s_{N_i\_start}$  and  $s_{N_i\_end}$  are the mileage of the beginning and the end of the range. Here, the spatial range affected by the anomaly is where the anomaly occurs.

*Time intensity:* Time intensity describes how long the anomaly affects the traffic in the influenced spatial region. This is determined by the duration of the anomaly. Let  $\beta$  be the spatial intensity, then,

$$\beta = \mu T \quad (2)$$

Here,  $T$  is the duration of the anomaly and  $\mu$  is a parameter capturing the recovery ability of the anomaly. This parameter is

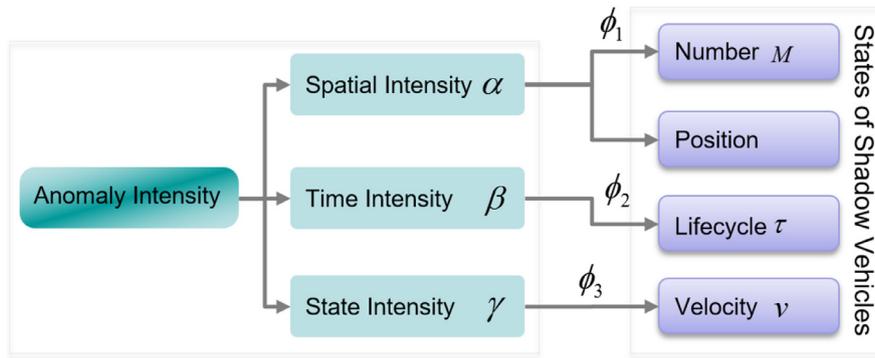


Fig. 3. Mapping between an anomaly and shadow vehicles.

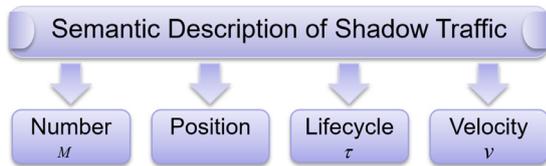


Fig. 4. The contents of shadow traffic's semantic description.

a constant and greater than or equal to 1; if the traffic flow will return to normal once the anomaly disappears, then  $\mu \equiv 1$ .

*State intensity:* State intensity gives the impact of the anomaly on traffic capacities and is determined by the ratio of the traffic capacity before the anomaly to the capacity after the anomaly. Considering that the anomaly may impact each neighboring lane differently, we define the state intensity of the anomaly as the union of state intensities over all lanes affected by the anomaly. Let  $\gamma$  be the state intensity of the anomaly,

$$\gamma = \bigcup_{i=0}^n \gamma_{N_i} = \bigcup_{i=0}^n \left\{ \frac{v_{N_i-acci}}{v_{N_i-free}} \right\} \quad (3)$$

Here  $\gamma_{N_i}$  describes the state intensity of the anomaly on lane  $N_i$  and  $v_{N_i-free}$  and  $v_{N_i-acci}$  are the traffic speeds before and after the anomaly on lane  $N_i$ .

3.2. Semantic description of shadow traffic

In our model, we assume that there are shadow traffic flows on the places where there has an anomaly. We use the states of shadow traffic to describe the behaviors of the traffic anomaly. There are many properties of shadow traffic. In our model, we

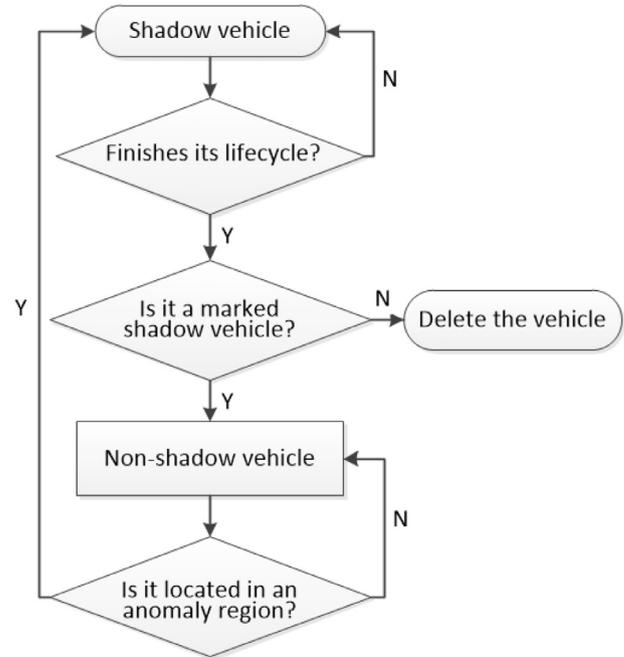


Fig. 6. The transformation between a shadow vehicle and a non-shadow vehicle.

model shadow traffic by the number of shadow vehicles, the positions of shadow vehicles, the lifecycles of shadow vehicles, and the velocities of shadow vehicles, as depicted in Fig. 4.

*Number:* We place shadow vehicles on the road segments where the anomaly is. Let  $M$  be the number of shadow vehicles placed for the anomaly.

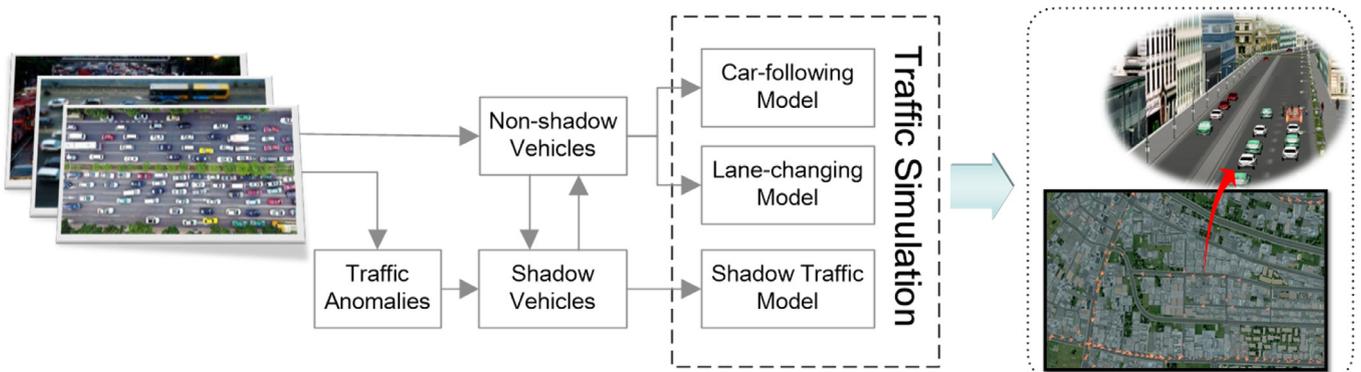


Fig. 5. Traffic simulation framework containing shadow vehicles.

**Table 1**  
Feature comparisons for traffic anomalies as reported by some sources.

Sources	Simulation method	Relation to traffic models	Convenience
VISSIM [24]	Parking space, etc.	Independent	No
SUMO [31]	Parking space, variable speed sign, etc.	Independent	No
Ours	Shadow traffic	Coupled	Yes

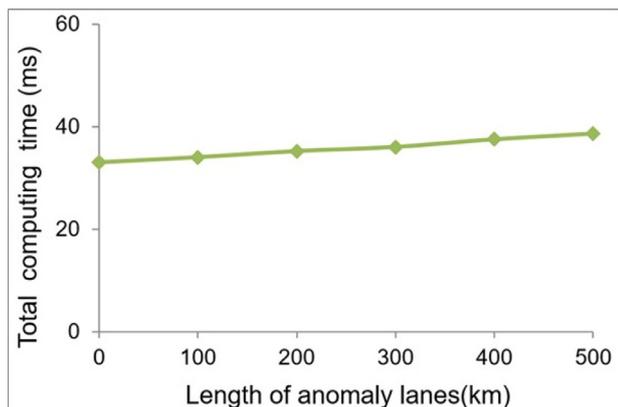


Fig. 7. The total compute time as a function of the length of anomaly lanes.

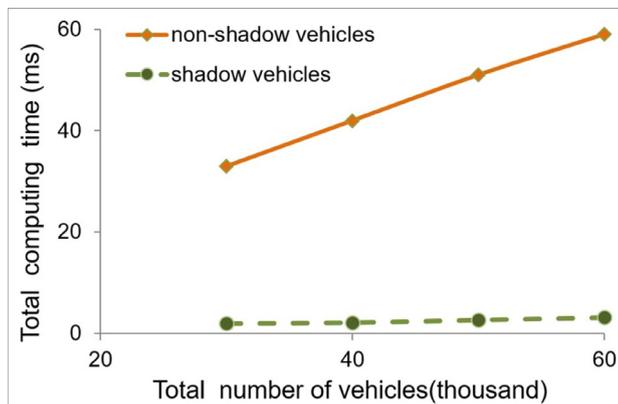


Fig. 8. The total compute time as a function of the number of non-shadow /shadow vehicles.



Fig. 9. Traffic simulation reacts to a fallen tree.

**Position:** The positions of these shadow vehicles describe where they are located respectively.

**Lifecycle:** The lifecycles of these shadow vehicles describe how long these vehicles participate in traffic simulations. The anomaly may not exist permanently. When it disappears, the lives of these shadow vehicles are over. Let the lifecycle of a shadow vehicle be  $\tau$ .

**Velocity:** These shadow vehicles should have their states in their lifecycles. We describe the state of a shadow vehicle by its velocity  $v$ .

### 3.3. Mapping between the anomaly and shadow traffic

The intention of mapping between the anomaly and shadow traffic is such that the behavior of the anomaly (i.e., its occurrence, life, and death) can be described through the occurrence, life, and death of shadow vehicles. Specifically, we transform the spatial intensity, the time intensity and the state intensity of the anomaly to the number of shadow vehicles, the positions of shadow vehicles, the lifecycles of shadow vehicles, and the velocities of shadow vehicles, as depicted in Fig. 3. The relationships between these values are as follows.

The number of these shadow vehicles  $M$  is determined by the spatial intensity of the anomaly  $\alpha$ . Let  $\varphi_1$  be the mapping function between  $\alpha$  and  $M$ ,

$$M = \varphi_1(\alpha) = \sum_{i=0}^n \left\lfloor \frac{|S_{N_i-end} - S_{N_i-start}|}{L} \right\rfloor \quad (4)$$

where  $L$  is the safety gap between two neighboring vehicles on a lane.

The positions of these shadow vehicles are the specific positions of the above  $M$  vehicles, which are also determined by the spatial intensity of the anomaly. Specifically, we place shadow vehicles at evenly spaced intervals of size  $L$  in the range that is determined by  $\alpha$ . Note that if there is already a vehicle, then we label it as a marked shadow vehicle. This label is only used to indicate that the shadow vehicle cannot disappear after its life is over. The lifecycle of a shadow vehicle describes how long the vehicle participates in traffic simulations. The value is determined by the time intensity of the anomaly  $\beta$ . Let the lifecycle be  $\tau$ , and the mapping function between  $\beta$  and  $\tau$  be  $\varphi_2$ . We assume that  $\tau$  is equal to  $\beta$ :

$$\tau = \varphi_2(\beta) = \beta \quad (5)$$

The velocity of a shadow vehicle describes its velocity in its lifecycle and is determined by the state intensity of the anomaly  $\gamma$ . Let the mapping function between  $\gamma$  and the velocity of the shadow vehicle  $v$  be  $\varphi_3$ ,

$$v = \varphi_3(\gamma) = v_{j\_free} * \gamma_j \quad (6)$$

where  $j$  is the ID number of the lane where the vehicle is located,  $j \in \bigcup_{i=0}^n N_i$ .  $v_{j\_free}$  is the traffic velocity on lane  $j$  before the anomaly, and  $\gamma_j$  is the state intensity on lane  $j$ .

## 4. Traffic simulations containing shadow vehicles

We model traffic anomalies using shadow vehicles and model traffic flow in the areas where there are no anomalies using non-shadow vehicles. Here we give a framework for a traffic simulation to model shadow vehicles and non-shadow vehicles, as shown in Fig. 5. We simulate shadow vehicles by our shadow traffic model. We simulate non-shadow vehicles by a car-following model and a lane-changing model. The transformation between a shadow vehicle and a non-shadow vehicle is shown in Fig. 6.

More specifically, the following steps describe how to realize a traffic simulation containing both shadow vehicles and non-shadow vehicles.



Fig. 10. Simulation snapshots for an anomaly caused by damaged lanes.

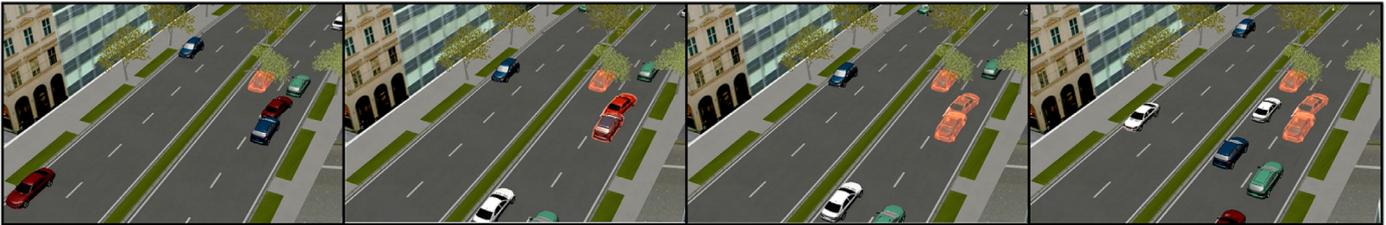


Fig. 11. Simulation snapshots for an anomaly caused by damaged lanes and a crash.



Fig. 12. Traffic simulation reacts to a combination of a damaged lane and a collision.



Fig. 13. Traffic simulation when some pedestrians cross a vehicle lane.

Step 1: If an anomaly occurs, generate the states of shadow vehicles, including the number, positions, lifecycles, and velocities, according to Section 3;

Step 2: Use different methods to update the states of non-shadow and shadow vehicles in every time step:

2.a: The states of a non-shadow vehicle are determined by the car-following model and the lane-changing model that the simulation adopts.

2.b: The states of a shadow vehicle in its lifecycle are determined by the intensity of the anomaly, which can be obtained from Section 3.3.

Step 3: If a shadow vehicle finishes its lifecycle on the current step, the number of shadow vehicles is reduced by one and the type of the vehicle is changed according to the following rules:

3.a: If the vehicle is a marked shadow vehicle, transform it into a non-shadow vehicle and update its states according to Step 2.a;

3.b: If the vehicle is an unmarked shadow vehicle, then delete the vehicle.

## 5. Results

### 5.1. Comparison study

We present a unified model for abnormal traffic behavior simulations using shadow traffic. We also show how shadow vehicles participate in traffic simulations. Detailed feature comparisons between some typical methods and our method are shown in Table 1. It is clear that in VISSIM [24] and SUMO [31] traffic simulation systems, users must generate and edit the anomaly indirectly by modifying parking spaces, speed signs, etc., which is complicated and separate from traffic models.

### 5.2. Performance analysis

We demonstrate the efficiency of our model through two experiments. In these experiments, we adopt the intelligent driver car-following model presented by Treiber et al. [32] and the all-in-one lane-changing model presented by Wang et al. [14] to model the non-shadow vehicles in the traffic simulation. The results were collected on a work station with an Intel(R) Core 8 Xeon(R) CPU E31240 @3.4 GHz and 4.0 GB RAM.



Fig. 14. Traffic simulation when these pedestrians finish their crossing.

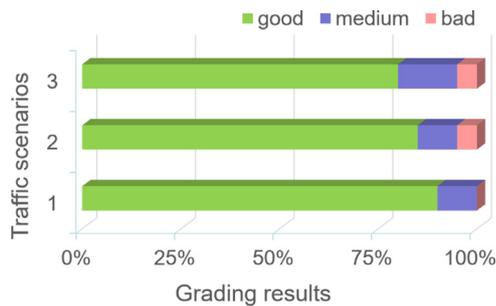


Fig. 15. The results of our user study. The scenarios of damaged lanes, combination of a damaged lane and a collision, and pedestrian and vehicle interaction are numbered 1–3, respectively.

### 5.2.1. Impact of anomaly regions

We model traffic anomalies by transforming them to the states of shadow vehicles. A larger anomaly range will introduce a larger number of shadow vehicles, and the number of shadow vehicles will affect the efficiency of the simulation.

There are nearly 30 thousand non-shadow vehicles in our simulation. Fig. 7 shows the increase in computation time with an increasing anomaly region. What can be seen from the chart is that the computing time negligibly increases as the anomaly region becomes larger. Specifically, there is only a few milliseconds of additional overhead despite the length of anomaly lanes is nearly 500 km. In fact, it is practically impossible for such large-scale anomalies to exist in a traffic simulation system.

### 5.2.2. Effect of the number of shadow vehicles

To better evaluate the performance of our model when more shadow vehicles are added in the simulation, we compare the computational time as a function of the number of non-shadow

/shadow vehicles. Fig. 8 shows the comparison result. It is clear that far less time is required to get the states of these shadow vehicles in the simulation compared to that of non-shadow vehicles.

### 5.3. Validation for traffic anomaly simulations

To demonstrate the benefits of our model, we use it to reconstruct some typical traffic scenarios, including those caused by damaged lanes, crashes between vehicles, combinations of both, pedestrian and vehicle interactions, etc.

**Damaged lanes:** This scenario depicts how traffic flow goes on after a roadside tree falls. We set one unmarked shadow vehicle at the position of the fallen tree, much like a real broken-down vehicle (Fig. 9). We then observe the upstream vehicles reacting to the anomaly. They slow down and change their lane to the neighboring lane – in agreement with what happens in real life. Fig. 10 shows some snapshots of the situation. It is also highlighted in our accompanying video.

**Combination of a damaged lane and a collision:** This scenario depicts the traffic behavior after a succession of traffic anomalies. A roadside tree falls causing a vehicle to change lanes to avoid the tree. This change of lanes causes a crash with another vehicle. We set one unmarked shadow vehicle at the point of the fallen tree first and then label the two marked vehicles as shadow vehicles (Fig. 12). It can be observed that the two-lane road is shut down a short time later. Fig. 11 shows some snapshots of the situation before and after the crash.

**Pedestrian and vehicle interaction:** In this scenario, we simulate the behavior of a traffic in which vehicles share space with pedestrians. This traffic pattern is called the “Bern Model”. In this model, the majority of pedestrian crossings are informally negotiated wherever they are needed. While pedestrians walk across roads, we set shadow vehicles at the points of the lanes where they cross (Fig. 13). These shadow vehicles will be deleted after the pedestrians finish their crossing (Fig. 14). Fig. 16 and Fig. 17 show some snapshots of the situation before and after the crossing. The outcome is highlighted in the supplementary video.

**Limitations:** Different from vehicle-pedestrian mixed flow model proposed by Chao et al. [28], in which they use a force-based crowd model, an agent-based traffic model and a gap acceptance criterion for mixed traffic simulations, we model mixed traffic through the introduction of shadow traffic intuitively and efficiently. However, the description of pedestrian and vehicle interactions is limited to the type in which vehicles give way to pedestrians in our model. In real life, there are some complex scenarios in which vehicles and pedestrians fight for positions. The combination of traffic simulations, crowd simulations, and our model may finally tackle this challenging problem.

For intuition, we list the simulation videos of these three traffic scenarios for a user study. We randomly choose 20 non-professionals. The video of each scenario is three minutes length approximately. These non-professionals watch each video respectively and then grade it according to their intuitions. There are



Fig. 16. Simulation snapshots for the generations of shadow vehicles in pedestrian and vehicle interactions.

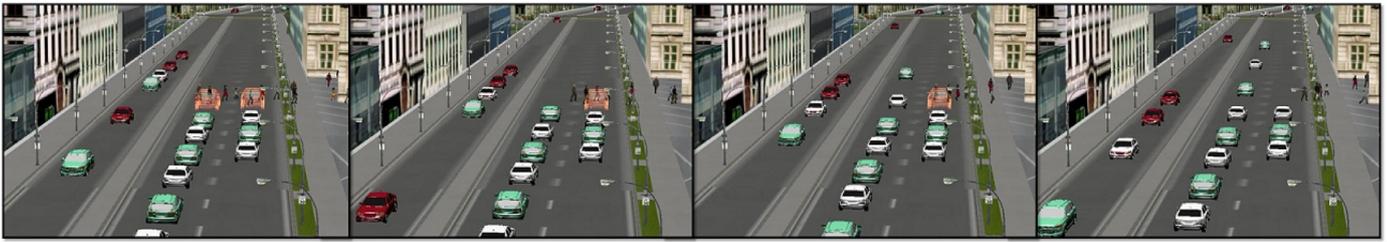


Fig. 17. Simulation snapshots for the disappearances of shadow vehicles in pedestrian and vehicle interactions.

three grades here: good, medium, bad. If the simulation result is in agreement with what happens in real life, the grade is good; If the simulation result is entirely different from what happens in real life, the grade is bad; Otherwise, the grade is medium. Fig. 15 gives the results. It shows that our simulation results are in agreement with what happen in real life to some degree.

## 6. Conclusions

We introduce a simple yet novel concept, shadow traffic, to model traffic anomalies with little additional computational overhead to traffic simulations. Through creative transformations of anomaly properties and integrations of shadow / non-shadow vehicles in a traffic simulation framework, a variety of traffic anomalies can be depicted in a unified way. Our model can well describe how the anomaly itself is evolving. It is complementary to most of existing traffic flow models and it is easy to integrate the model into almost all kinds of traffic simulation systems. In the future, we would like to build a real-time editing traffic anomaly simulation system, in which users can introduce and edit anomalies whenever and wherever they need in traffic simulations. We also would like to explore modeling of complex vehicle-crowd interactions.

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## Supplementary material

Supplementary material associated with this article can be found, in the online version, at [10.1016/j.cag.2017.07.004](http://dx.doi.org/10.1016/j.cag.2017.07.004).

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